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THE PITTSBURGH SYMPOSIUM ON GROUP THEORY AND QUANTUM MECHANICS

In connection with the Annual Meeting of the Society at Pittsburgh, it was arranged to hold a symposium on *Group theory and quantum mechanics* under the joint auspices of the American Mathematical Society and the American Physical Society. The program was planned in co-operation with Professors H. P. Robertson and J. H. Van Vleck.

The symposium was held on Saturday afternoon, December 29, in the Union Room of the Carnegie Institute of Technology, and was followed by an interesting discussion; Professor Robertson presided. There was a large attendance of mathematicians and physicists. Abstracts* of the several addresses appear below.

I. *Representations and ray-representations in quantum mechanics*, by Professor John von Neumann.

It is known that any symmetry property of an intuitively described physical system finds its mathematical expression in the existence of a certain group under the operations of which the mechanical determining equations of the system are invariant. This group-theoretical principle of symmetry has been particularly suitable for group-theoretical methods. The mathematical theory, the application of which has in this connection led to many results in various fields of quantum physics, is the so-called *theory of representations*. As the states of the quantum mechanical system are really described, not by a uniquely defined wave-function, but by one which is only known—and has a physical meaning only—to within a constant factor of absolute value 1, the discussion must be based not on the theory of representations proper, but on that of representations to within a constant factor of absolute value 1. These have been called ray-representations. If the group of symmetry operations is a continuous (Lie) group, the representations may be discussed by considering the so-called infinitesimal representations of the group. These are characterized by certain commutation properties which, in the case of a ray-representation, will only be valid to within an additive constant multiple of unity. In some cases, these extra terms can be eliminated (transformed away). In some other cases they vanish automatically. There exist groups, however, for which neither is the case and the extra terms play an essential role. These possibilities are discussed for various main types of symmetry-groups.

* Because of illness, Professor von Neumann was unable to read the paper which he had prepared. It is nevertheless desirable to include the abstract of his work in this report for it furnishes a natural introduction to the other papers and the notions contained therein were referred to by the other speakers.

II. *Symmetry relations in various physical problems*, by Professor E. P. Wigner.

Because of the linearity of the quantum mechanical equations, the operations of displacement and rotation in space-time must be represented by linear operators. These operators carry over the wave function of a state in a certain frame of reference into the wave function of the same state in another frame of reference. They form a representation of the group of possible coordinate transformations, which is the inhomogeneous Lorentz group. If these transformations are known, the equation of motion for the system can be derived from them, because they tell in particular how the state is changed by a shift of the time scale. Thus there is a unique correspondence between the possible Lorentz invariant equations of quantum mechanics, on the one hand, and the representations of the inhomogeneous Lorentz group by linear operators, on the other.

The gain in generality as compared with the usual tensor calculus consists in that no assumption concerning the field nature of the equations is made. Thus one must obtain in the present calculus all equations, even such (if they exist) in which the coordinate is quantized, for example.

The irreducible representations of the inhomogeneous Lorentz group were determined. There are, mathematically, three cases, according to whether $p_t^2 - p_x^2 - p_y^2 - p_z^2$ is positive, zero, or negative, where the p's represent the infinitesimal displacement operators. The first case corresponds to a positive rest mass; in this case, it can be proved that all the representations may be interpreted as tensor equations. The same is true for the last case, which, however, has probably no significance. In the second case, which corresponds to photons, a new class of representations enters, which falls into the half-reducible class.

The detailed discussion will appear elsewhere in a joint paper with P. A. M. Dirac, who first perceived this problem.

III. *Some applications of group theory to non-relativistic physical problems*, by Professor J. H. Van Vleck.

Some groups encountered in non-relativistic quantum mechanics are the permutation group, the free rotation group, and crystallographic groups. The utility of the permutation group is greatly diminished by the fact that only a small fraction of the irreducible representations are compatible with the Pauli exclusion principle. A nice application of the free rotation group is to the calculation of the intensity distribution in the multiple structure of quadrupole lines, which are forbidden by the ordinary dipole selection rules, but are of importance in astrophysics. Two important applications of the crystallographic groups are to the theory of magnetism, and to the theory of the small vibrations of the nuclei about their equilibrium positions in polyatomic molecules. If the molecule has symmetry elements, then as E. P. Wigner and E. B. Wilson, Jr., have both shown, the theory of the characters and irreducible representations of the crystallographic group tells us how many normal vibrations should coincide in frequency, and which should be active in infra-red absorption or in the Raman effect. Also it permits a factorization of the secular equation which materially reduces the labor of calculating the fundamental frequencies.

IV. *Some applications of group theory to Dirac's relativistic theory*, by Professor Gregory Breit.

1. Dirac's relativistic theory introduces four matrices α_μ, $(\mu=1, 2, 3, 4)$, which satisfy the equation $\alpha_\mu\alpha_\nu+\alpha_\nu\alpha_\mu=2\delta_{\mu\nu}$. According to a general theorem given by P. Jordan and E. Wigner by means of the theory of finite group representations there exists one and only one irreducible representation of these by means of finite matrices which is of interest for Dirac's equation. As shown by Pauli, this fact suffices for the proof of the relativistic invariance of Dirac's equation. The commutation relations of the α_μ may be thus held responsible for the success of Dirac's equation as a relativistic description of the electron.

2. Using the matrices Γ^1, Γ^2, Γ^3, $\Gamma^4=\alpha_4\alpha_1$, $\alpha_4\alpha_2$, $\alpha_4\alpha_3$, $-i\alpha_4$ and x^1, x^2, x^3, $x^4=x$, y, z, ict it is possible to find, for every Lorentz transformation $x'^\nu=b_\mu{}^\nu x^\mu$ a matrix γ such that $\gamma^{-1}\Gamma^\nu\gamma=b_\mu{}^\nu\Gamma^\mu$. It was shown by von Neumann that these representations of the Lorentz group are capable of being represented, for pure space-time rotations, in two independent two-dimensional subspaces of the wave function components ψ_μ. This is essentially due to the existence of a matrix Γ with eigenwerte $+1$, $+1$, -1, -1 $(\Gamma=\Gamma^1\Gamma^2\Gamma^3\Gamma^4)$ which commutes with all γ which represent pure rotations. Taking linear combinations of the ψ_μ which correspond to the eigenaxes of Γ, one has two-dimensional representations of the proper Lorentz group. For transformations involving reflections Γ, γ do not commute and a four-dimensional representation is necessary. The two-dimensional representations are now known as spinor representations.

Except for differences of notation the spinor treatment is already contained in Weyl's discussion of Dirac's equation where the isomorphism of the unimodular group of linear transformations of two variables with the group of Lorentz transformations was made use of and the difference between pure rotations and reflections was clearly set forth.

3. By methods of group theory or by the direct elegant approach of Dirac in his book one can discuss possible forms of relativistic interaction energy. Among the possible forms with four component equations only the Dirac and Pauli forms and their combinations are possible under the extended Lorentz group. For pure rotations other forms such as the one proposed by Rojansky are available. By the same methods one can discuss relativistically possible forms of proton, neutron, electron, neutrine interactions. One has the following possibilities without introducing derivatives of wave functions:

$$-F_0(\Psi^*\Psi)+F_1(\Psi^*\alpha_1\Psi)+F_2(\Psi^*\alpha_2\Psi)+F_3(\Psi^*\alpha_3\Psi)$$
$$-E_0(\Psi^*\rho_1\Psi)+E_1(\Psi^*\sigma_1\Psi)+E_2(\Psi^*\sigma_2\Psi)+E_3(\Psi^*\sigma_3\Psi)$$

$$F_0=\phi_2\psi_1-\phi_1\psi_2+\phi_3\psi_4-\phi_4\psi_3,\quad F_1=-\phi_2\psi_4+\phi_4\psi_2+\phi_1\psi_3-\phi_3\psi_1,$$
$$E_0=\phi_4\psi_1-\phi_2\psi_3-\phi_3\psi_2+\phi_1\psi_4,\quad E_1=\phi_2\psi_2-\phi_2\psi_4-\phi_1\psi_1+\phi_3\psi_3,$$

etc., with Dirac's original matrices as well as corresponding combinations with $-(\psi^*\phi)$, $\psi^*\vec{\alpha}\psi$ and $-(\psi^*\rho_1\,\phi)$, $\psi\vec{\sigma}\phi$. Here Ψ refers to the heavy particle, ϕ and ψ to the two types of light particles. The first form is Fermi's and the other forms are alternative possibilities.

J. R. KLINE,
Associate Secretary

THE FEBRUARY MEETING IN NEW YORK

The three hundred twentieth meeting of the American Mathematical Society was held at Columbia University on Saturday, February 23, 1935. The attendance included the following 115 members of the Society:

E. B. Allen, R. L. Anderson, R. G. Archibald, Max Astrachan, N. H. Ball, Louis Baron, Samuel Beatty, M. F. Becker, Garrett Birkhoff, H. F. Bohnenblust, C. B. Boyer, Richard Brauer, H. W. Brinkmann, A. B. Brown, J. H. Bushey, R. H. Cameron, A. D. Campbell, W. R. Church, J. A. Clarkson, George Comenetz, Richard Courant, J. L. Doob, Jesse Douglas, R. L. Echols, H. T. Engstrom, T. S. Fiske, W. B. Fite, D. A. Flanders, M. M. Flood, M. C. Foster, R. M. Foster, Jeanette Fox, T. C. Fry, B. P. Gill, S. L. Greitzer, C. C. Grove, Marshall Hall, Morris Halperin, G. G. Harvey, Alan Hazeltine, Einar Hille, Lulu Hofmann, Harold Hotelling, J. C. Hughes, E. M. Hull, M. H. Ingraham, W. H. Ingram, Nathan Jacobson, E. R. van Kampen, Edward Kasner, S. C. Kleene, J. R. Kline, A. W. Landers, M. K. Landers, Solomon Lefschetz, D. H. Lehmer, R. R. Lyle, L. A. MacColl, H. F. MacNeish, R. S. Martin, H. H. Mitchell, Deane Montgomery, C. N. Moore, F. J. Murray, D. S. Nathan, C. A. Nelson, Emmy Noether, E. P. Northrop, Alta Odoms, H. B. Phillips, H. A. Rademacher, W. C. Randels, H. W. Raudenbush, M. S. Rees, F. G. Reynolds, Moses Richardson, R. G. D. Richardson, J. F. Ritt, S. L. Robinson, H. A. Ruger, M. F. Schmeiser, C. E. Seely, Stefan Serghiesco, Max Shiffman, J. A. Shohat, C. G. Shover, L. P. Siceloff, L. G. Simons, M. E. Sinclair, James Singer, Abraham Sinkov, L. L. Smail, E. D. Smith, A. A. Stafford, J. D. Tamarkin, E. W. Titt, C. B. Tompkins, A. W. Tucker, I. L. Van Dyck, Oswald Veblen, J. L. Walsh, R. M. Walter, Morgan Ward, S. E. Warschawski, M. S. Webster, M. J. Weiss, W. A. Wilson, Clement Winston, Aurel Wintner, H. P. Wirth, Jack Wolfe, E. W. Woolard, Oscar Zariski, Leo Zippin, Max Zorn.

The meeting opened on Saturday morning with two sectional sessions, one of analysis and one of algebra, arithmetic, and geometry. There was a general session in the afternoon at which Professor Morgan Ward gave an address on *Arithmetic theory of linear recurring series.*

There was no meeting of the Council or of the Trustees.

Titles and cross references to the abstracts of the papers read at this meeting follow below: papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 17 were read before the section of analysis, Professor Einar Hille presiding; those numbered 18 to 30 before the section of algebra, arithmetic, and geometry, Professor Brinkmann presiding; those numbered 31 to 61 at the general session, Pro-

fessors Solomon Lefschetz and J. D. Tamarkin presiding. Mr. Fialkow was introduced by Professor Kasner, Dr. Martin by Professor Bochner, Mr. Netzorg by Professor Bohnenblust, Dr. Offord by Professor Tamarkin, Professor Fekete by Professor Tamarkin, and Mr. Whiteman by Professor Mitchell.

1. *A generalization of harmonic functionals*, by Dr. F. G. Dressel. (Abstract No. 41-1-82-*t*.)

2. *Fourier series convergence criteria, as applied to continuous functions*, by Dr. J. A. Clarkson and Dr. W. C. Randels. (Abstract No. 41-1-81.)

3. *On an approximate functional equation of Paley*, by Dr. W. C. Randels. (Abstract No. 41-1-92.)

4. *$(n-1)$-dimensional characteristic strips and Cauchy's problem for partial differential equations*, by Dr. E. W. Titt. (Abstract No. 41-3-103.)

5. *Non-commutative integration*, by Mr. Garrett Birkhoff. (Abstract No. 41-3-104.)

6. *Linear differential equations with almost periodic coefficients*, by Dr. R. H. Cameron (National Research Fellow). (Abstract No. 41-3-105.)

7. *The dynamics of Gramme-ring generators* (preliminary report), by Mr. W. H. Ingram. (Abstract No. 41-1-87.)

8. *On linear transformations in $\mathfrak{L}_p$, $p>1$*, by Mr. F. J. Murray (National Research Fellow). (Abstract No. 41-3-106.)

9. *Linear difference equations with arbitrary real spans*, by Dr. W. T. Martin (National Research Fellow). (Abstract No. 41-3-107.)

10. *A new approximation method in solving linear differential equations with non-oscillating coefficients*, by Professor Cornelius Lanczos. (Abstract No. 41-3-108-*t*.)

11. *On the angular derivatives of univalent functions*, by Dr. S. E. Warschawski. (Abstract No. 41-3-109.)

12. *On positive Cotes' numbers*, by Mr. D. L. Netzorg. (Abstract No. 41-3-110.)

13. *On Fourier transforms*. III, by Dr. A. C. Offord. (Abstract No. 41-3-111-*t*.)

14. *Converse theorems of summability for Dirichlet series*, by Professor Otto Szász. (Abstract No. 41-3-112-*t*.)

15. *On convergence in length*, by Professor C. R. Adams and Dr. Hans Lewy. (Abstract No. 41-3-113-*t*.)

16. *Orthogonal polynomials and functions of the second kind*, by Professor J. L. Walsh. (Abstract No. 41-3-114-*t*.)

17. *Some generalizations of Paley's theorems on Fourier series with positive coefficients*, by Professor M. Fekete. (Abstract No. 41-3-115-*t*.)

18. *Congruences with a common middle envelope*, by Professor Malcolm Foster. (Abstract No. 41-3-116.)

19. *Trajectories and lines of force*, by Mr. Aaron Fialkow. (Abstract No. 41-1-83-*t*.)

20. *Finiteness of the number of quadratic fields with even discriminant and Euclid algorithm* (preliminary report), by Miss Jeanette Fox. (Abstract No. 41-3-117.)

21. *A set of defining relations for the simple group of order* 1092, by Dr. Abraham Sinkov. (Abstract No. 41-1-94.)

22. *On Lucas's test for the primality of Mersenne's numbers*, by Dr. D. H. Lehmer. (Abstract No. 41-3-118.)

23. *Representation of Lie algebras*, by Dr. Nathan Jacobson. (Abstract No. 41-1-89.)

24. *Divisibility sequences of third order*, by Mr. Marshall Hall. (Abstract No. 41-1-85.)

25. *Proper reduction of regular matric polynomials*, by Mr. M. M. Flood. (Abstract No. 41-1-84.)

26. *Note on metric geometry*, by Professor I. J. Schoenberg. (Abstract No. 41-3-119-*t*.)

27. *On the Poincaré group of plane algebraic curves* (first communication), by Professor Oscar Zariski. (Abstract No. 41-3-120.)

28. *A note on the word mantissa*, by Dr. C. C. Grove. (Abstract No. 41-3-121.)

29. *The classification and general solution of certain diophantine problems which involve special systems of equations of the second degree*, by Professor H. A. Simmons. (Abstract No. 41-3-122-*t*.)

30. *On Bieberbach's theory of cubic constructions*, by Professor Raymond Garver. (Abstract No. 41-3-123-*t*.)

31. *On convergence factors in multiple series summable by Nörlund means*, by Professor C. N. Moore. (Abstract No. 41-3-124.)

32. *A new inversive invariant*, by Professor Edward Kasner. (Abstract No. 41-3-125.)

33. *On mechanical quadratures*, by Professor J. A. Shohat. (Abstract No. 41-3-126.)

34. *Pseudo-covariants in a Galois field and their algebraic and geometric applications*, by Professor A. D. Campbell. (Abstract No. 41-3-127.)

35. *On the law of quadratic reciprocity*, by Mr. Albert Whiteman. (Abstract No. 41-1-99.)

36. *Studies in the summability of Fourier series by Nörlund means*, by Mr. Max Astrachan. (Abstract No. 41-3-128-*t*.)

37. *Abstract continuous groups*, by Mr. Garrett Birkhoff. (Abstract No. 41-3-129-*t*.)

38. *On the lattice theory of ideals*. II, by Mr. Garrett Birkhoff. (Abstract No. 41-1-75-*t*.)

39. *An algebraic proof for the quasi-congruence order of the* R_n, by Dr. L. M. Blumenthal (National Research Fellow). (Abstract No. 41-1-76-*t*.)

40. *A short proof of a theorem of K. Menger*, by Dr. L. M. Blumenthal (National Research Fellow). (Abstract No. 41-1-77-*t*.)

41. *On certain analytic continuations and analytic homeomorphisms*, by Dr. A. B. Brown. (Abstract No. 41-1-78-*t*.)

42. *On the locus of an analytic equation in the real plane*, by Dr. A. B. Brown. (Abstract No. 41-1-79-*t*.)

43. *Note on the clock problem in relativity*, by Professor J. W. Campbell. (Abstract No. 41-1-80-*t*.)

44. *Involutorial transformations associated with a rational ruled surface*. Part I, by Dr. L. A. Dye. (Abstract No. 41-3-130-*t*.)

45. *Divisibility sequences of fourth order*, by Mr. Marshall Hall. (Abstract No. 41-1-86-*t*.)

46. *Rational methods in the theory of Lie algebras*, by Dr. Nathan Jacobson. (Abstract No. 41-1-88-*t*.)

47. *Some theorems concerning certain spaces of R. L. Moore*, by Mr. F. B. Jones. (Abstract No. 41-1-90-*t*.)

48. *The dual of Lie's contact transformation of dynamics*, by Professor Edward Kasner. (Abstract No. 41-3-131-*t*.)

49. *Osculating turbines*, by Professor Edward Kasner. (Abstract No. 41-3-132-*t*.)

50. *A note on the analysis of variance*, by Dr. Solomon Kullback. (Abstract No. 41-3-133-*t*.)

51. *On samples from a multivariate normal population*, by Dr. Solomon Kullback. (Abstract No. 41-1-91-*t*.)

52. *A basis for residual polynomials in n variables*, byProfessor Marie Litzinger. (Abstract No. 41-3-134-*t*.)

53. *Operational methods in theoretical population problems*, by Professor T. H. Rawles. (Abstract No. 41-3-135-*t*.)

54. *A certain three-dimensional continuum*, by Dr. W. T. Reid. (Abstract No. 41-3-136-*t*.)

55. *The relative connectivities of symmetric products* (preliminary report), by Mr. Moses Richardson. (Abstract No. 41-1-93-*t*.)

56. *On the coefficients of a typically-real function*, by Dr. M. S. Robertson (National Research Fellow). (Abstract No. 41-3-137-*t*.)

57. *Generalization of two theorems of Hardy and Littlewood on power series*, by Professor Otto Szász. (Abstract No. 41-3-138-*t*.)

58. *Matrices permutable with the rational canonical form of a matrix*, by Dr. Richard Trott. (Abstract No. 41-1-95-*t*.)

59. *On the laws of distribution of certain statistics for samples of n from Poisson's first law of probability*, by Professor F. M. Weida. (Abstract No. 41-1-96-*t*.)

60. *Abstract theory of inversion of finite sums*, by Professor Louis Weisner. (Abstract No. 41-1-97-*t*.)

61. *Some properties of prime-power groups*, by Professor Louis Weisner. (Abstract No. 41-1-98-*t*.)

J. R. KLINE,
Associate Secretary

THE APRIL MEETING AT STANFORD UNIVERSITY

The three hundred twenty-first meeting of the Society was held at Stanford University on Saturday, April 6, 1935. The meeting was called to order at 10:30 A.M., and was presided over at different times by Professors Blichfeldt and Bateman. There was a morning and an afternoon session.

Some forty people were present, including the following twenty-four members of the Society:

H. M. Bacon, Harry Bateman, B. A. Bernstein, H. F. Blichfeldt, F. A. Butter, C. E. Corbin, G. C. Evans, E. C. Goldsworthy, O. G. Harrold, E. R. Hedrick, R. D. James, D. N. Lehmer, S. H. Levy, C. F. Luther, J. C. C. McKinsey, W. A. Manning, A. D. Michal, C. B. Morrey, S. L. Parker, T. M. Putnam, Pauline Sperry, J. V. Uspensky, A. R. Williams, B. C. Wong.

Luncheon for members and their guests was served at the Stanford Union.

The titles of papers read at the meeting follow. Those whose abstract numbers are followed by the letter *t* were read by title. Mr. P. O. Bell and Miss Manning were introduced by Professor Manning; Mr. Webb was introduced by Professor Bell; Messrs. Elconin, Paxson, Highberg, and Taylor were introduced by Professor Michal; in the absence of Mr. Webb and Mr. Elconin their papers were read by Professor Michal.

1. *Tetrahedra associated with canonical developments for the equation of a curved surface*, by Mr. P. O. Bell. (Abstract No. 41-3-144.)

2. *The problem of the shopper*, by Professor Harry Bateman. (Abstract No. 41-3-145.)

3. *Generation of an n-valued logic by one binary operation*, by Mr. D. L. Webb. (Abstract No. 41-3-146.)

4. *A theorem relating the differentials of Fréchet and Gateaux*, by Mr. V. Elconin. (Abstract No. 41-3-161.)

5. *A reduced set of postulates for Riemannian differential geometry in abstract vector spaces*, by Professor A. D. Michal. (Abstract No. 41-3-148.)

6. *Postulates for Boolean algebra involving the operation of complete disjunction*, by Professor B. A. Bernstein. (Abstract No. 41-3-163.)

7. *On geometry of numbers*, by Professor H. F. Blichfeldt. (Abstract No. 41-3-155.)

8. *Sets of independent postulates for linear spaces and vector spaces*, by Mr. I. E. Highberg and Mr. A. E. Taylor. (Abstract No. 41-3-159.)

9. *The geometry of abstract euclidean spaces*, by Mr. I. E. Highberg, Professor A. D. Michal, and Mr. A. E. Taylor. (Abstract No. 41-3-158.)

10. *On simply transitive permutation groups with transitive Abelian subgroups of the same degree*, by Miss Dorothy Manning. (Abstract No. 41-3-154.)

11. *New results for the number $g(n)$ in Waring's problem* (preliminary report), by Dr. R. D. James and Mr. H. S. Zuckerman. (Abstract No. 41-3-162.)

12. *A simple proof of Tschebysheff's inequalities*, by Professor J. V. Uspensky. (Abstract No. 41-3-156.)

13. *A simplification in the theory of weak convergence*, by Professors H. E. Bray and G. C. Evans. (Abstract No. 41-3-165.)

14. *Correspondences connected with a pencil of n-ics*, by Professor A. R. Williams. (Abstract No. 41-3-164.)

15. *A note on Taylor's theorem*, by Professor A. F. Moursund. (Abstract No. 41-3-149-*t*.)

16. *A new solution of the Gauss problem on $h(s^2d)/h(d)$*, by Professor Gordon Pall. (Abstract No. 41-3-150-*t*.)

17. *Configurations inscriptible in a plane cubic curve*, by Dr. J. M. Feld. (Abstract No. 41-3-151-*t*.)

18. *A detail in Kronecker's program*, by Professor E. T. Bell. (Abstract No. 41-3-152-*t*.)

19. *An abstract triplet metric space* (preliminary report), by Mr. E. W. Paxson. (Abstract No. 41-3-153-*t*.)

20. *Primitive functions on a finite set*, by Mr. V. Elconin. (Abstract No. 41-3-147-*t*.)

21. *A fallacy of the so-called osculatory interpolation formulas*, by Professor C. H. Forsyth. (Abstract No. 41-3-157-*t*.)

22. *A set of independent postulates for a metric space*, by Mr. A. E. Taylor. (Abstract No. 41-3-160-*t*.)

T. M. PUTNAM,
Associate Secretary

SHORTER NOTICES

A Mathematical Treatise on Vibrations in Railway Bridges. By C. E. Inglis. Cambridge, University Press, and New York, Macmillan, 1934. 203 pp. and 65 figures.

Professor Inglis has written a book which, notwithstanding its particular application to vibrations in railway bridges, should prove a valuable asset to both the mathematician and engineer interested in the general subject of vibration theory. For his treatment the author has used the method of harmonic analysis throughout, a method not only powerful, but particularly suited for handling vibrations in structures due to complicated distributions of moving loads. The rapid convergence of the series involved in the analysis permits many useful approximations which are in excellent agreement with the unwieldy, exact formulas.

In the introductory chapter the reader is acquainted with the principles involved in the methods to be used. There is also interesting reading on the behavior of railway bridges subject to moving loads. The first two chapters serve to illustrate the method of harmonic analysis by applications to beams subject to various end restraints and vibrating under various types of periodic forces. In the following eight chapters the principles are applied to railway bridges, each chapter treating the problem more generally than the previous one, and finally culminating in a study of the complex modes of vibration arising when locomotive spring movement is included in the general picture. The vector method for finding particular solutions of the linear differential equations involved is next demonstrated. This graphical method, extremely useful and capable of easy application, is rather infrequently used by the mechanical engineer, although it is quite familiar to most electrical engineers. Short-span bridges of 50 feet or less are treated in a separate chapter, and locomotive and bridge characteristics are fully discussed. In the final chapter the author determines the dynamic bending moments and shearing forces due to the vibrations, pointing out the important differences which result if the usual method of differentiating the curve of bending the proper number of times is employed for this calculation.

Although many complex problems are treated exactly in the book, practically every case is also considered approximately and clearly demonstrated with typical numerical examples.

The descriptive matter is written in an interesting manner and the text is sufficiently illustrated. The practising engineer will find extremely useful a synopsis at the very end of the book in which the most important results are collected and re-stated.

The reviewer recommends this book highly both to the student of vibrations interested in the mathematical analysis peculiar to structures vibrating under various types of moving loads and to the engineer interested in railway bridge design.

L. C. PESKIN

Carrés Magiques au Degré n. By Général E. Cazalas. Paris, Hermann, 1934. 191 pp.

It is a relief to find a book or memoir on the subject of magic squares which is not a mere collection of specimens with no attempt at unification and analysis. This work by General Cazalas is an attempt, and a very successful one, to bring these interesting and baffling forms into some sort of unified theory.

The author has undertaken to put on a solid analytical basis the results announced in 1906 by G. Tarry who, by the use of what he called "numeral series," was able to arrive at results of astonishing generality in constructing squares characterized not only by the fact that the sum of the elements in any row or column or in the two main diagonals should be constant, but also by the further property that the sum of the squares and the sum of the cubes, etc., of the elements in any of these lines should also be constant. Thus, for example, the square of order 9 which follows is not only magic in the ordinary sense, but the sum of the squares in any of the lines is also constant.

0	64	47	14	75	31	25	62	42
34	17	78	36	19	56	50	3	67
59	39	22	70	53	6	72	28	11
69	52	8	74	27	10	58	41	21
13	77	30	24	61	44	2	63	46
38	18	55	49	5	66	33	16	80
48	4	68	35	15	79	37	20	54
73	29	9	57	40	23	71	51	7
26	60	43	1	65	45	12	76	32

To illustrate the method of "numeral series" we give a square of order 5 constructed with the "keys" (11) and (32).

00	11	22	33	44
32	43	04	10	21
14	20	31	42	03
41	02	13	24	30
23	34	40	01	12

The symbol ab stands for $5a+b$, a and b taking values from 0 to 4. In ordinary notation the square would read:

0	6	12	18	24
17	23	4	5	11
9	10	16	22	3
21	2	8	14	15
13	19	20	1	7

This is a magic square of the ordinary sort, using, however, the numbers from 0 to $n-1$ instead of the numbers from 1 to n as is customary.

In the construction it is seen that the elements in any horizontal row are obtained from the one on the left by the addition of the numbers of the "key," multiples of 5 being thrown out as they arise. Similarly the elements in any column are obtained from the one above by addition of the numbers of the

second "key," multiples of 5 again being thrown out as they arise. This method, applicable to any square of prime order, will, however, not always work when the order of the square is composite. The squares obtained by it are more easily obtained by the "uniform step" method, a complete theory of which has been worked out in a paper apparently not known to the author published in the Transactions of this Society in 1930 (vol. 31, No. 3, pp. 529–551).

Although the method of "numeral series" is of no great importance for the problem of ordinary squares, it yields extraordinary results when applied to "hypermagic squares," that is, squares which are magic in the squares of the elements or in the cubes of the elements. Thus for squares of order p^2, squares magic in the squares of the elements are obtained, and trimagic squares are also obtained, that is, magic in the cubes of the elements. The smallest bimagic square given is of order 8 and the smallest trimagic square of order 128. The method of construction does not seem to indicate whether bimagic squares of order less than eight are possible. It is not difficult to prove that squares of order less that 7 can not be bimagic. Whether there are bimagic squares of order 7 or not is in doubt, and the reviewer hopes to answer that question before long.

D. N. LEHMER

Einführung in die Differentialrechnung und Integralrechnung. By Edmund Landau. Groningen-Batavia, P. Noordhoff N. V., 1934. 368 pp.

The reviewer's task of reporting on this book is considerably simplified by the fact that it has already been reviewed in the American Mathematical Monthly (Nov., 1934). Presumably Professor Ritt's review has been seen by most of the readers of the Bulletin, but the novelty of this work seems to justify some further comment.

The book is confined solely to the analytical parts of calculus; neither are there any problems for the student to work, nor are there the familiar applications to geometry and mechanics. The author's aim is to present the theory "in exacter und lückenloser Weise." Your reviewer will not appraise the author's success in this endeavor by searching for slips in rigor, but will confine his remarks to points likely to be of interest to the teacher of calculus.

A knowledge of the fundamental rules of algebra is presupposed. These are set forth in the introduction, which also contains the notion of the Dedekind cut, the necessary theorems on finite and infinite sets of numbers, and a proof of the existence of $a^{1/n}$ for $a \geqq 0$ and n integral. This last, however, is used only for $n=2$.

Chapter 1 contains the usual definitions and theorems on the limit of a sequence. Since the student is supposed to know merely the meaning of a^n for n a positive or negative integer or zero and the existence of $a^{1/n}$ for $a \geqq 0$, it is necessary in Chapter 2 to develop the whole theory of logarithms and exponentials. We find $\log x$ defined as $\lim_{n\to\infty} k(x^{1/k}-1)$, where $k=2^n$. The existence and usual properties of logarithms are then deduced. The constant e is now defined as the solution of $\log y=1$, e^x as the solution of $\log y=x$, and $a^x=e^{x\log a}$ for $a>0$. The usual theory of exponentials readily follows with unusual brevity.

Chapters 3, 4, and 8 deal with functions, continuity, and limits of functions.

Continuity of $f(x)$ at $x=a$ is defined in the usual ϵ, δ fashion. The concept $\lim_{x\to a} f(x)$ is defined by first proving that there is at most one value η, such that the function $g(x)$ is continuous at a, where $g(x)=f(x)$ for $x\neq a$ and $g(x)=\eta$ for $x=a$. If such an η exists, then $\eta=\lim_{x\to a} f(x)$.

Differentiation for the functions so far defined is discussed in the next three chapters. The formula $dy/dx=(dy/du)(du/dx)$, almost always incorrectly proved, here has a correct proof. The troublesome theorem on the limit of a function of a function is avoided by introducing the auxiliary function $g(h)=[f(a+h)-f(a)]/h$ for $h\neq 0$, $g(h)=0$ for $h=0$, which is continuous at $h=0$ and simplifies this and other proofs. The terms maximum and minimum, and increasing and decreasing at a point are defined and the few theorems possible at this point are proved.

Chapters 9–11 cover Rolle's theorem, the mean value theorem, and the extended mean value theorem, with applications to maximum and minimum values, indeterminate forms, and the theorem that, if $f(x)$ is continuous and $f'(x)=0$ in ab, then $f(x)=f(a)$. The next four chapters discuss series, uniform convergence, and power series, including the theorem on the termwise differentiation of power series. By means of this last and an ingenious use of the theorem mentioned before, the ordinary expansions of e^x and $(1+x)^\alpha$ are shown to be correct. The usual expansion by Taylor's series is not given except in the form of a theorem applicable only to functions defined as sums of power series.

In Chapter 16, $\sin x$ and $\cos x$ are defined as sums of power series, and $\tan x$ and $\operatorname{ctn} x$ as quotients of $\sin x$ and $\cos x$. The formulas for differentiating $\sin x$ and $\cos x$ come at once from the theorem on termwise differentiation of power series, without the use of $\lim_{x\to 0}(\sin x/x)$. By the mean-value theorem $\cos x$ is shown to have a least positive zero, which is defined as $\pi/2$. Then formulas for $\sin(x+y)$ and $\cos(x+y)$ are derived by means similar to those used in proving the expansion of e^x in power series. The remaining formulas follow at once. The remainder of Part I consists of chapters on partial derivatives, inverse and implicit functions, inverse trigonometric functions, and the resolution of rational functions into partial fractions.

In Part II, on integral calculus, there are fewer novelties and the student of real functions is on familiar ground. The first four chapters contain the definition of indefinite integrals, general theorems, and sufficient technique to show the integrability of rational functions and of some irrational functions. The next three chapters discuss definite integrals, based on the Riemann definition, and give the usual theorems, including termwise integration of series. Two chapters on improper integrals follow and the book closes with short chapters on the Gamma function and Fourier series. As in Part I there is an entire absence of drill work and of applications.

What shall we say is the chief value of this book? It is clear that it could not serve as a text for the American college student of the present time. Nevertheless it should exert a considerable influence on teachers of mathematics and it should be required reading for writers of textbooks in calculus. Nothing is more needed for the effective teaching of this subject than a textbook which is rigorous as well as simple, and such books are almost non-existent. The chief value of this work, to the writer's mind, is that it shows that rigorous proofs

are simple proofs and that the theory is to be used, instead of being laid away on a shelf. Let this be a stimulus to the textbook writer, but let him beware of slavish copying. For we must build on what the student has already learned in the secondary school, and it is questionable whether the topsy-turvy treatment of logarithms, exponentials, and trigonometric functions here given could be "put across" to the average beginner. Finally it must not be forgotten that the most difficult parts of calculus to make rigorous and at the same time simple and concise are those applications to geometry and mechanics which Professor Landau eschews. There is still work to be done.

W. A. WILSON

Introduction to General Topology. By W. Sierpinski. Translated from the Polish by C. Cecilia Krieger. The University of Toronto Press, 1934. 235 pp.

Topology, long considered as more or less outside the pale of "normal" mathematics, is at last coming into its own. It has passed in the last few years from the pure "mémoire" to the book stage. The number of treatises on diverse phases of the subject is still restricted enough. We can only welcome therefore the present volume by the leader of the Polish school, and thank the translator for having made it available to the non-Polish mathematical public.

Professor Sierpinski's treatise is of modest proportions, and deals exclusively with the abstract space side of the question. It is the work of a "purist" and shows little trace of the steady drift towards combinatorial topology on the part of those particularly interested in the applications of the subject, or in its contacts with other branches of mathematical science. The postulates are carefully defined, their inter-relations investigated in detail and with thoroughness. The author has very wisely chosen the class of open sets as fundamental in defining topological spaces, a procedure amply justified by experience. Without indulging in a description of topics, we may mention that metrization is fully treated, but dimensionality barely touched upon at the end.

This is the second volume of a more extensive series probably projected by the eminent author, the first of which (translated into French) dealt with transfinite numbers. Are we to hope for a third volume, which, as the program would seem to indicate, would begin with the Urysohn-Menger theory?

SOLOMON LEFSCHETZ

NOTES

The Italian Society for the Progress of Science announces the following subjects for its Mussolini prize in the mathematics section: Finite thermoelastic determinations; Characteristics of differential systems; Waves of discontinuity, in particular, waves caused by shocks, and the theory of explosives. Competition is restricted to members of the Society and closes August 31, 1935.

The gold medal of the Royal Astronomical Society has been awarded to Professor E. A. Milne, Rouse Ball professor of mathematics at the University of Oxford, for his work on radiactive equilibrium and the theory of stellar atmospheres.

Dr. Moritz v. Rohr, of the University of Jena, has been awarded a Leibniz medal for his work in geometrical optics.

Dr. Charles A. Kraus, professor of chemical research at Brown University, has been awarded the Willard Gibbs medal of the Chicago section of the American Chemical Society for 1935, for his research on the theory of solutions.

The following have been elected to membership in the Royal Society of London: E. N. da C. Andrade, Quain professor of physics, University of London; J. S. Foster, professor of physics, McGill University; and R. S. Stoneley, lecturer in mathematics, University of Cambridge.

The following ninety-four doctorates, with mathematics or mathematical physics as major subject, were conferred during 1934 in universities in the United States and Canada; the major subject is mathematics unless otherwise specified. The university, month in which the degree was conferred, minor subject (other than mathematics), and title of dissertation are given in each case if available.

E. B. Allen, Rensselaer Polytechnic Institute, June, minor in physics, *Extension of a theorem due to M. D'Ocagne to partial differential equations in n variables.*

A. E. Andersen, Harvard, June, *Topics in the theory of binary forms.*

Frances E. Baker, Chicago, March, *A contribution to the Waring problem for cubic functions.*

D. H. Ballou, Harvard, June, *A class of completely monotonic functions every positive power of which is also completely monotonic.*

J. L. Barnes, Princeton, June, *On the Laplace-Stieltjes transformation* I.

J. J. Barron, Wisconsin, June, *The application of asymptotic forms to an expansion problem of the Sturm-Liouville type where the coefficient of the parameter changes sign.*

Miriam F. Becker, Yale, June, *On relative fields.*

A. F. Bernhard, Michigan, June, *The mechanics of the top.*

Solomon Bilinsky, Washington University (St. Louis), June, minor in physics, *A theory of functions of an abstract variable.*

M. T. Bird, Illinois, June, minors in statistics and physics, *On generalizations of sum formulas of the Euler-Maclaurin type.*

W. M. Borgman, Chicago, August, *Intersector varieties in hyperspace.*

F. L. Brooks, Ohio State, August, *Various properties of the metric limit.*

G. M. Brown, Michigan, February, *On sampling from compound populations.*

J. A. Clarkson, Brown, June, *Riemann-Stieltjes double integrals and bounded variation for functions of two variables.*

Nancy Cole, Harvard, June, *The index form associated with an extremaloid.*

Byron Cosby, II, Chicago, March, *Fields for multiple integrals in the calculus of variations.*

G. F. Cramer, Missouri, August, *Rings and ring-ideals in a relative quadratic field.*

R. H. Downing, West Virginia, June, minor in mathematical physics, *Flat-sphere geometry in non-euclidean n-space.*

T. L. Downs, Jr., Harvard, June, *On the planar points of an analytic surface.*

Mary J. Fisher, Toronto, March, *A new method in the theory of algebraic functions of one variable.*

Amelia Frank, Wisconsin, October, major in physics, *The effect of crystalline fields on the magnetic susceptibilities of* $SM+++$, *and* $EU+++$, *and the heat capacity of* $SM+++$.

B. C. Getchell, Michigan, June, *Integration of interval functions.*

H. A. Giddings, Massachusetts Institute of Technology, June, minor in chemistry, *On the extension of the notion of developable surfaces to* V_2 *and* V_3 *in* R_4.

Margaret Gurney, Brown, June, *Some general existence theorems for partial differential equations of hyperbolic type.*

Mary E. Haller, Washington, June, minor in physics, *Self-projective rational octavics.*

E. K. Haviland, Johns Hopkins, June, *On the theory of absolutely additive distribution functions.*

M A. Heaslet, Stanford, June, minor in physics, *Concerning the development coefficients of an aequianharmonic function.*

Grace M. Hopper (Mrs. V. F.), Yale, June, *New types of irreducibility criteria.*

Olive Hughes, Bryn Mawr, June, minor in physics, *A certain mixed linear integral equation.*

Antoinette Killen Huston (Mrs. R. E.), Chicago, June, *The integral bases of all quartic fields with a group of order eight.*

J. A. Hyden, Cornell, June, minor in physics, *The Weddle and Kummer surfaces for restricted positions of six base points.*

Nathan Jacobson, Princeton, January, *Non-commutative polynomials and cyclic algebras.*

Evan Johnson, Jr., Chicago, August, *Dynamics of variable masses.*

H. S. Kaltenborn, Michigan, June, *On Stieltjes mean integrals.*

Rosella Kanarik, Pittsburgh, June, *Fundamental regions in* S_4 *for the Hessian group.*

Nathan Kaplan, Massachusetts Institute of Technology, June, minor in

physics, *A study of three-dimensional Riemann surfaces in six-dimensional euclidean space.*

Gertrude S. Ketchum (Mrs. P. W.), Illinois, June, minor in English, *On certain generalizations of the Cauchy-Taylor expansion theory.*

Irving Kittell, New York University, June, minor in physics, *On the classification and coloring of spherical maps.*

S. C. Kleene, Princeton, January, *A theory of positive integers in formal logic.*

J. C. Knipp, Pittsburgh, June, *Frégier surfaces.*

R. L. Krueger, Marquette, June, minor in physics, *Plane quintic curves, a classification and projective construction.*

W. S. Lawton, Pennsylvania, February, *A method of deriving a homogeneous linear differential equation of the second order satisfied by a certain class of orthogonal polynomials.*

Marjorie Leffler, Ohio State, August, *A lemma in potential theory.*

Jack Levine, Princeton, June, *Projective scalar differential invariants.*

F. A. Lewis, Ohio State, August, *Some properties of an infinite class of collineation groups.*

Marie Litzinger, Chicago, August, *A basis for residual polynomials in n variables.*

A. N. Lowan, Columbia, May, *Application of the Laplace transformation to certain problems in physics.*

S. T. Ma, California (Berkeley), May, *The relations between the solutions of the linear differential equations of the second order having four regular singular points.*

L. A. MacColl, Columbia, May, *On the distribution of the zeros of sums of exponentials of polynomials.*

C. W. MacGregor, Pittsburgh, June, minors in physics and mechanics, *Selected problems in the theories of flat plates and plane stress.*

J. D. Mancill, Chicago, December, *The minimum of a definite integral with respect to unilateral variations.*

M. F. Manning, Massachusetts Institute of Technology, June, major in physics, *Exact solutions of the Schrödinger equation.*

W. T. Martin, Illinois, June, minor in astronomy, *On expansions in terms of a certain general class of functions.*

B. B. Murdock, Yale, June, *Skew frequency curves.*

E. P. Northrop, Yale, June, *An operational solution of the Maxwell field equations.*

Rufus Oldenburger, Chicago, March, *Composition and rank of n-way matrices and multilinear forms.*

J. W. Querry, State University of Iowa, February, minors in applied mathematics and education, *A general theory of mechanical quadrature.*

W. C. Randels, Brown, June, *Some problems in the theory of Fourier series.*

J. A. F. Randolph, Cornell, September, minor in astronomy, *Carathéodory measure and a generalization of the Gauss-Green lemma.*

Louis Rauch, California (Berkeley), August, minor in physics, *The almost periodic solution of a problem in forced vibrations by a process of iteration.*

R. F. Rinehart, Ohio State, August, *Some properties of the discriminant matrices of a linear associative algebra.*

M. I. S. Robertson, Princeton, June, *On the theory of univalent functions.*

W. J. Robinson, Ohio State, August, *Rings with one prime.*

J. B. Rosser, Princeton, June, *A mathematical logic without variables.*

M. F. Rosskopf, Brown, June, *Some inequalities for non-uniformly bounded ortho-normal polynomials.*

J. M. Rowat, Toronto, June, *Existence theorems for the differential equation* $dy/dx=f(x, y)$ *under conditions of Perron integrability.*

O. K. Sagen, Chicago, June, *The integers represented by sets of positive ternary quadratic non-classic forms.*

G. E. Schweigert, Johns Hopkins, June, *The analysis of certain curves by means of derived local separating points.*

Nathan Schwid, Wisconsin, June, minor in applied mathematics, *The asymptotic forms of the Hermite and Weber functions.*

C. H. W. Sedgewick, Brown, June, *Generalized Lambert series.*

Robert Serber, Wisconsin, June, major in physics, *Some optical properties of molecules.*

J. A. Sharpe, Wisconsin, June, major in physics, *Surface motion in the compressional phase of a deep-focus earthquake and the effects of a layered crust.*

Sister Ann Elizabeth Shea, Wisconsin, October, *Regular Cremona transformations in* S_4.

R. C. Shook, Chicago, June, *Concerning Waring's problem for sixth powers.*

A. J. Smith, Pennsylvania, June, *Concerning upper semi-continuous collections on curves and two-dimensional manifolds.*

F. C. Smith, Michigan, June, *Contributions to the study of the asymptotic developments of analytic functions.*

F. H. Steen, Harvard, June, *On a class of polynomials which minimize definite integrals.*

J. K. Stewart, West Virginia, June, minor in mathematical physics, *Some birational transformations associated with a class of ruled varieties in n-space.*

J. W. T. Suckau, Ohio State, June, *Uniform convergence and the Lebesgue theory.*

Sister Helen Sullivan, Catholic University, June, minor in physics and mechanics, *The non-self-symmetric quadrilaterals in- and circumscribed to the rational cuspidal quartic with a line of symmetry.*

W. R. Talbot, Pittsburgh, August, *Fundamental regions in* S_6 *for the simple quaternary* G_{60}, *type* I.

Henrietta P. Terry, Illinois, June, minor in chemistry, *Abelian subgroups of order* P^m *of the I-groups of the Abelian groups of order* P^n *type* 1, 1, 1, $\cdots$.

R. W. Thomas, Pittsburgh, June, minor in physics, *Four-space representation of chain congruences in the plane of two complex variables.*

G. R. Trott, Johns Hopkins, June, *On the canonical form of a non-singular pencil of Hermitian matrices.*

P. L. Trump, Wisconsin, October, *On a reduction of a matrix by the group of matrices commutative with a given matrix.*

J. L. Vanderslice, Princeton, April, *Non-holonomic geometries.*

Henry Van Engen, Michigan, June, *On the asymptotic behavior of analytic functions.*

J. I. Vass, Wisconsin, minor in applied mathematics, *A class of boundary problems of highly irregular type.*

W. J. Wagner, Pittsburgh, June, minor in astronomy, *Reciprocals of some elementary figures with respect to a space cubic.*

R. J. Walker, Princeton, June, *Reduction of the singularities of an algebraic surface.*

G. C. Webber, Chicago, June, *Waring's problem for cubic functions.*

K. W. Wegner, Wisconsin, June, *The equivalence of pairs of Hermitian matrices.*

C. B. Wright, Pittsburgh, June, *Fundamental regions in S_4 for the collineation group $G_{2.360}$.*

C. R. Wylie, Jr., Cornell, June, minor in physics, *Curves whose tangents belong to a linear complex.*

The following doctorates were conferred in 1933 but not included in the list in the preceding volume of this Bulletin (vol. 40, pp. 378–381):

F. G. Dressel, Duke, June, *A boundary value problem for the heat equation.*

Mabel Griffin, Duke, June, *Invariants of pfaffian systems.*

Dr. W. N. Bailey has been appointed Richardson lecturer in pure mathematics at the University of Manchester.

Mr. W. V. D. Hodge has been elected to a fellowship and appointed lecturer and director of mathematical studies at Pembroke College, Cambridge.

The retirement is announced of Professor Alfred Pringsheim of the University of Munich.

Dr. Franz Rellich, of the University of Göttingen, has been appointed professor of mathematics at the University of Marburg.

Dr. Emanuel Sperner has been appointed professor of mathematics at the University of Königsberg.

Professor W. H. Cramblet has been elected president of Bethany College.

Dr. J. L. Doob, of Columbia University, has been appointed associate professor of mathematics at the University of Illinois.

The Trustees of Columbia University, at their meeting on March 4, accepted the resignation of Dr. T. S. Fiske as professor of mathematics, to take effect at the end of the present academic year, and voted to bestow upon him the title of professor emeritus of mathematics. After his release from active service he will retain a study and office at Columbia University.

Professor Philip Franklin, of the Massachusetts Institute of Technology, has obtained leave of absence for advanced study and foreign travel during the coming academic year.

Professor Arthur Haas, of the University of Vienna, will be visiting profes-

sor of physics for ten weeks during the winter of 1936 at Drexel Institute. During his stay there Dr. Haas will be available for lectures in neighboring institutions.

Professor Rudolph E. Langer, of the University of Wisconsin, has been appointed lecturer in mathematics in Harvard University for the year 1935–36.

Professor W. R. Longley, of Yale University, has been nominated as representative of the Mathematical Association of America on the National Research Council for a three-year term beginning July 1, 1935. He succeeds Professor H. L. Rietz of the University of Iowa.

Dr. E. L. Mackie has been promoted to a professorship at the University of North Carolina.

Dr. I. J. Schoenberg has been appointed acting assistant professor of mathematics at Swarthmore College.

Professor Stephen Timoshenko, of the engineering mechanics department of the University of Michigan, has been appointed Hitchcock Professor at the University of California at Berkeley for the second semester of the year 1934–35 and has lectured there under that foundation.

Associate Professor J. L. Walsh has been appointed professor and tutor of mathematics at Harvard University.

Dr. Hassler Whitney has been appointed assistant professor and tutor of mathematics at Harvard University, and has been granted leave of absence for the first semester of the academic year 1935–36.

Associate Professor D. V. Widder, of Harvard University, has been granted a leave of absence for the year 1935–36.

Professor Norbert Wiener, of the Massachusetts Institute of Technology, has been granted leave of absence for the next academic year during which time he will be research professor of mathematics at the National Tsing Hua University in Peking, China.

The following appointments to instructorships are announced: College of the City of New York (tutor, evening session): Dr. R. C. Shook; West Virginia University: Dr. R. H. Downing, Dr. J. K. Stewart.

Dr. Karl Bopp, professor of mathematics at the University of Heidelberg is dead.

Arthur Lionel Pedder, formerly mathematical tutor at Magdalen College, Oxford, died December 15, 1934, at the age of sixty-six.

The death is reported of Dr. Gustav Ritter v. Escherich, professor of mathematics at the University of Vienna.

The death is reported of Dr. Arthur Schidlof, professor of mathematics at the University of Geneva.

Dr. E. A. Partridge, of the West Philadelphia Boys' High School, died March 22, 1934. He had been a member of the American Mathematical Society since 1894.

Professor M. I. Pupin, emeritus professor of electromechanics at Columbia University, died March 12, 1935 at the age of seventy-seven.

Professor E. B. Skinner, of the University of Wisconsin, died April 3, 1935. He had been a member of this Society since 1899. A more adequate notice will appear in a later issue of this Bulletin.

Professor Emmy Noether, of Bryn Mawr College, died April 14, 1935. A more adequate notice will appear in a later issue of this Bulletin.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

184. Professor B. P. Gill: *Generalization of Lucas' u_n, v_n for recurrences of third and higher order.* Preliminary report.

Initial values are given for k solutions of a homogeneous recurrence relation of order k in such a way that the solutions have algebraic properties that generalize those of the functions of Lucas. For example, the coefficients in the addition theorems expressing the functions with subscript $m+n$ as bilinear forms in those with subscripts m and n are semi-invariants of the generating polynomial of the recurrence. (Received March 22, 1935.)

185. Professor Leonard Carlitz: *On a certain function connected with polynomials in a Galois field.*

This paper continues the study of the function defined in a previous paper (abstract 39-5-161, this Bulletin, vol. 39 (1933), p. 352). Applications are here made to the properties of a class of polynomials analogous to the cyclotomic polynomials; an application of a different nature is made to the theory of the congruence $t^P - t \equiv A \pmod{P}$, where A and P are polynomials. (Received March 23, 1935.)

186. Dr. Max Zorn: *The automorphisms of Cayley's non-associative algebra.* Preliminary report.

A normal form for the automorphisms of the Cayley numbers is established, by means of a 7-dimensional vector calculus; as a corollary we obtain a result of Brandt concerning the numbers a defining automorphisms by $x \to a^{-1}xa$. Furthermore we get a rational representation of all automorphisms by six iterated transformations with suitable Cayley numbers. (Received March 14, 1935.)

187. Mr. Marshall Hall: *Indices in cubic fields.*

Dedekind first showed that one cannot always find an algebraic integer of unit index in a given field, by exhibiting fields in which there is an index divisor common to all integers of the field. In particular the greatest common index divisor in a cubic field is at most two. This paper investigates the question: is there an algebraic integer in an arbitrary field whose index is the greatest

common divisor of all indices of the field? The answer is in the negative. From a study of the forms which represent the indices, it is shown that there exist cubic fields in which the least index is arbitrarily large. (Received March 20, 1935.)

188. Dr. G. C. Webber (National Research Fellow): *Concerning transcendental numbers.*

Gelfond and Schneider have proved, independently, that in general ω^α is transcendental where both ω and α are algebraic numbers. By simple generalizations of Schneider's proof we obtain results such as the following: if ω is transcendental, θ_1 and θ_2 linearly independent non-rational algebraic numbers, one of ω^{θ_1}, ω^{θ_2} is transcendental. (Received March 21, 1935.)

189. Dr. H. S. M. Coxeter and Dr. Abraham Sinkov: *The groups determined by the relations* $S^l = T^m = (S^{-1}T^{-1}ST)^p = 1$.

The usual approach to the problem of groups generated by two operators has been to assign the periods of the generators and the period of their product. The present paper studies a slightly different approach in which the period of the product of the two generators is replaced by the period of their commutator. A method is presented for representing these groups geometrically; the representation is three-dimensional and has for its fundamental region a tetrahedron with dihedral angles $2\pi/l$, $2\pi/m$ at two opposite edges, the four remaining dihedral angles being $\pi/2p$. The space required is spherical, euclidean, or hyperbolic according as $\sin(\pi/l)\sin(\pi/m)$ is greater than, equal to, or less than $\cos(\pi/2p)$. The group is generated by rotations about the two special edges of the tetrahedron and is finite only when the space is spherical. A complete solution is given for all the spherical and euclidean groups; in the latter cases it is necessary to impose an additional defining relation upon the generators. This new condition determines the period of their product and suggests that the two different methods of approach be combined in the study of the hyperbolic groups. In this connection, it is shown that the four relations $S^l = T^m = (ST)^n = (S^{-1}T^{-1}ST) = 1$ are in general insufficient to determine a finite group. (Received March 22, 1935.)

190. Dr. Nathan Jacobson: *Locally compact totally disconnected rings.*

This paper is concerned with an investigation of locally compact, separable, totally disconnected rings F. We have divided it into two parts. In Part I we determine the structure of the additive group of a simple ring F (not necessarily associative or commutative). The results are as follows: If the characteristic $\chi(F) = p$, F is additively a direct sum of cyclic groups of order p. If $\chi(F) = 0$, F is additively a finite vector space over the field of p-adic numbers K_p. It follows in this case that F is a hypercomplex system with a finite basis over K_p. In Part II we restrict ourselves to associative fields F (not necessarily commutative). The methods here are arithmetic in nature. We prove the existence of a unique maximal compact and open subring R of F. By means of R a *valuation* (Bewertung) of F is defined. F is then shown to be a cyclic algebra

over its centrum. This centrum may be determined as, indeed, it has been by v. Dantzig and by Hasse and Schmidt. (Received March 12, 1935.)

191. Mr. W. R. Church: *Theory of finite distributive free structures.*

A structure is a system of elements in which equality and two binary operations, cross-cut and union, are defined and for which the usual axioms (see Dedekind, Werke, II, pp. 236–271) are postulated. This paper is concerned with structures which satisfy the distributive laws for these operations. The existence of the free distributive structure generated by means of n elements is shown by obtaining a necessary and sufficient condition for the independence of n elements, and then giving an explicit realization of this condition by means of certain elements in a finite Boolean algebra of minimal dimension. Among other things this approach furnishes the means whereby some details of the description of the structures of this class are investigated. (Received March 23, 1935.)

192. Mr. Garrett Birkhoff: *Postulate systems for Lie groups.*

It is shown by an elementary argument that if in some neighborhood L of the origin of ordinary n-space there is defined an associative law of composition $c_t=\phi^i(a;b)$ of class C' such that $\phi^i(a;0)=\phi^i(0;a)=a_i$, then L is the nucleus of a Lie group. Related arguments lead to interesting topologically invariant postulate systems for Lie groups. (Received March 21, 1935.)

193. Professor Edward Kasner: *The duals of velocity families and natural families.*

A velocity family, in two dimensions, is defined by a differential equation of the form $y''=(A+By')(1+y'^2)$. The author obtains a dual type in hessian line coordinates $v''=A+Bv'$. In this case the osculating cycles (oriented circles) along any straight line touch another line, and this constitutes a complete characterization of the dual type. In a velocity family the osculating circles at a point have a second point in common. Natural families constitute a subtype, Characterized by a certain relationship between the functions A and B, which yields a certain hyperosculating property (see Kasner, Princeton Colloquium, Chapter 2). The dual sub-type, obtained from certain infinitesimal contact transformations, discussed in a previous paper read at the February meeting (see abstract 41-3-131), is also characterized geometrically. (Received March 19, 1935.)

194. Dr. S. B. Myers (National Research Fellow): *Connections between differential geometry and topology.* II: *Closed surfaces.*

In a previous paper (see abstract 41-1-40) the author introduced the notion of the locus of "minimum points" on a complete analytic 2-dimensional Riemannian space and studied this locus on simply connected surfaces. In this paper the locus is studied on a closed (compact) Riemannian surface S. A point M on a geodesic ray g issuing from A is said to be a minimum point with

respect to A if M is the last point on g such that AM furnishes an absolute minimum (proper or improper) to the length of arcs joining A to M. The locus m of such points with respect to A is shown to be a continuous curve which is locally a tree; if S is analytic, m is a finite linear graph. The cyclomatic number of m equals the connectivity number mod 2 of S. The order of a point M of m as a vertex of m equals the number of shortest geodesics joining M to A. The end points of m are conjugate to A and are cusps of the locus of first conjugate points to A. The surface S is reduced to a single 2-cell a with m as its singular boundary; a is simply covered (except at A) by the geodesic rays through A cut off at their intersections with m. Thus a solution is given to the hitherto vaguely answered question as to when the field formed by the geodesic rays through A breaks down. (Received March 22, 1935.)

195. Professor H. L. Black: *Systems of curves and surfaces related under cyclic involutions.*

A class of involutions I_n of prime order, on algebraic curves or surfaces with but a finite number of invariant points, is studied. These exist only on loci having particular moduli; special cases have been examined previously by Hutcherson. It is shown in S_2 that all the invariant curves factor. We apply the term "related" to systems of invariant surfaces in S_3 which enable us to map the involution in turn on distinct loci in a common hyperspace $\sum_{(n^2+6n+11)/8}$. By this means correspondences $(1, m)$ one way and $(1, k)$ the other are established between loci in hyperspaces of different dimensions. (Received March 12, 1935.)

196. Professor J. H. Roberts: *Collections filling a plane.*

The author has shown that there exists an upper semi-continuous collection G filling the plane such that every element of G is a bounded continuum of diameter greater than 1 which does not separate the plane. In the present paper it is proved that there exists such a collection G each element of which is an arc. In view of a theorem of R. L. Moore this result may be stated as follows: there exists a continuous single-valued transformation T of the plane into itself such that for each point y the set of all points x for which $T(x)=y$ is an arc of diameter greater than 1. (Received March 22, 1935.)

197. Dr. D. C. Lewis (National Research Fellow): *The formal theory of conservative transformations in $2n$-dimensional space.*

A transformation carrying a point (x) in $2n$-dimensional space into a point (x') is said to be conservative if there exists a set of functions $X_i(x)$, such that the skew-symmetric determinant $|\partial X_i/\partial x_j-\partial X_j/\partial x_i| \neq 0$ and such that $\sum_{i=1}^{2n} X_i(x')dx_i'-\sum_{i=1}^{2n} X_i(x)dx_i$ is an exact differential, the unprimed x_i being the independent variables. The paper gives a formal theory of invariant points generalizing the work of Birkhoff who treated the case $n=1$ (Acta Mathematica, vol. 43). Analyticity of the transformation and the X's is assumed. (Received March 15, 1935.)

198. Mr. I. N. Kagno: *Irreducible non-toral graphs.*

Irreducible graphs, which cannot be mapped on a torus are found, corresponding to Kuratowski's irreducible non-planar graphs (Fundamenta Mathematicae, vol. 15 (1930), pp. 271–282). It is shown that there are a large number of such graphs, falling into eleven classes, and that no such graph has more than twenty-two, nor fewer than eight vertices. Among the preliminary theorems it is shown that: (1) if a graph when mapped on an orientable surface of genus p fails to separate that surface, then it can be mapped on a surface of lower genus; (2) a necessary and sufficient condition that a graph be on a surface of least possible genus is that the graph separate the surface into 2-cells, no matter how mapped on the surface. The existence of irreducible graphs which cannot be mapped on an orientable surface of any assigned genus is established. (Received March 8, 1935.)

199. Mr. Nelson Dunford: *Integration in general analysis.*

The known relationships between convergence almost everywhere, approximate convergence, and almost uniform convergence hold when the functions have as domain a metric space and as range a complete linear vector space (Banach's type B), provided, of course, the existence of a measure function is assumed. Starting with a class S_0 of uniformly continuous and bounded functions of the above type and defining a norm in terms of the Riemann Stieltjes integral $\|f\| = \int_E \|f(P)\| d\beta$, where β is the total variation of the measure function, we show that to every class of equivalent Cauchy sequences of functions in S_0 corresponds uniquely (except for a set of measure zero) a function which we call summable. The limit of the Riemann integrals of the functions forming a Cauchy sequence defines the integral of a summable function. This space of summable functions is complete. The integral is a completely additive and absolutely continuous function of sets. This formulation of the theory of integration is equivalent to that given by Bochner (Fundamenta Mathematicae, vol. 20). (Received March 21, 1935.)

200. Professor C. N. Moore: *On the multiplication of series summable by Nörlund means.*

Cesàro's theorem regarding the summability of the Cauchy product of two series which are convergent or summable by arithmetic means of a given order was one of the first contributions to the modern theory of summable series. This theorem can be generalized to the case of Nörlund means in the following fashion. Given two power series $\sum a_n z^n$, $\sum b_n z^n$, converging within the unit circle, we set $c_n = a_0 b_n + \cdots + a_n b_0$. Any series $\sum u_n$ will be said to be summable $(N; d)$ to U if $\sum_{i=0}^{i=n} s_i d_{n-i} / \sum_{i=0}^{i=n} d_i$, where $s_n = \sum_{i=0}^{i=n} u_i$, approaches U as a limit as n becomes infinite. We may now establish the following theorem: if $\sum u_n$ is summable $(N; a)$ to U and $\sum v_n$ is summable $(N; b)$ to V, where the methods $(N; a)$ and $(N; b)$ satisfy the conditions of regularity, then the Cauchy product of the two series will be summable $(N; c)$ to UV. This theorem includes Cesàro's result and some of its later generalizations as special cases. (Received March 22, 1935.)

201. Professor Salomon Bochner: *Summation of double Fourier series by circles.*

If $\sum a_{m,n}e^{i(mx+ny)}$ is the Fourier series of a double periodic function $f(x, y)$ of class L, we denote by $S_N(x, y)$ the sum of those terms of its series for which $m^2+n^2 \leqq N^2$. In every point of continuity (in a generalized sense) the sequence $S_N(x, y)$ is summable $(R, 1/2+\epsilon)$, for any $\epsilon>0$; and a corresponding result holds for functions of more than two variables. (Received March 23, 1935.)

202. Professor I. M. Sheffer: *A differential equation for Appell polynomials.*

Let $\{P_n(x)\}$ be a *set* of polynomials; that is, an infinite sequence in which P_n is of degree not exceeding n. There exist infinitely many linear differential equations of infinite order, of type $\sum L_n(x)y^{(n)}(x)=\lambda y(x)$, which for appropriate characteristic values $\{\lambda=\lambda_n\}$ have the sequence $\{P_n\}$ as solutions. Here L_n is a polynomial of degree not exceeding n. This type of differential equation is, then, a *universal form* for sets of polynomials. A natural question arises: what equations of this type are characteristic of certain known polynomial sets? In the present note the question is answered for Appell polynomials. There is obtained a peculiarly simple equation that characterizes Appell sets. (Received March 18, 1935.)

203. Mr. Garrett Birkhoff: *On the lattice theory of linear dependence.*

It is shown that the theory of "matroids" developed by Whitney to describe the abstract properties of linear dependence can be put into correspondence with a type of lattice (in fact, complemented semi-modular lattices), and that this relates certain properties of matroids to properties discovered in other connections. (Received March 21, 1935.)

204. Professor Alonzo Church and Dr. J. B. Rosser: *Some properties of conversion.*

The process of *conversion* is defined by Church in the Annals of Mathematics, (2), vol. 33 (1932), p. 357. A *normal form* of a formula is defined by Kleene in the Annals of Mathematics, (2), vol. 35 (1934), p. 535. Let a *reduction* be defined as a conversion which consists of applications of Rules I and II only and contains one and only one application of Rule II. It is proved that, if a formula has a normal form, the normal form is unique to within applications of Rule I, and any sequence of reductions of the formula must, if continued, terminate in the normal form. Also that if a formula B, not in normal form, is obtainable by conversion from a formula A, not in normal form, then, whether or not A and B have a normal form, it is possible to find a formula C which is obtainable by a sequence of reductions from A and likewise from B. Analogous theorems are proved about certain other processes which are similar in their character to conversion. (Received March 22, 1935.)

205. Professor Alonzo Church: *An unsolvable problem of elementary number theory.* Preliminary report.

Following a suggestion of Herbrand, but modifying it in an important respect, Gödel has proposed (in a set of lectures at Princeton, N. J., 1934) a definition of the term *recursive function*, in a very general sense. In this paper a definition of *recursive function of positive integers* which is essentially Gödel's is adopted. And it is maintained that the notion of an effectively calculable function of positive integers should be identified with that of a recursive function, since other plausible definitions of effective calculability turn out to yield notions which are either equivalent to or weaker than recursiveness. There are many problems of elementary number theory in which it is required to find an effectively calculable function of positive integers satisfying certain conditions, as well as a large number of problems in other fields which are known to be reducible to problems in number theory of this type. A problem of this class is the problem to find a complete set of invariants of formulas under the operation of conversion (see abstract 41-5-204). It is proved that this problem is unsolvable, in the sense that there is no complete set of effectively calculable invariants. (Received March 22, 1935.)

206. Dr. F. G. Dressel: *Green's theorem for generalized harmonic functionals.*

The paper presents a more general Green's theorem than that given in a preceding paper entitled *A generalization of harmonic functionals.* (Received March 15, 1935.)

207. Mr. Nelson Dunford: *On a theorem of Plessner.*

Assuming $f(x)$ to be (1) of period 2π and (2) of bounded variation and (3) that the total variation in t of $f(t+u)-f(t)$ tends to zero with u, Plessner (Journal für Mathematik, vol. 160 (1929), pp. 26–32) has shown that $f(t)$ is absolutely continuous (also proved by Wiener and Young, Transactions of this Society, vol. 35 (1933), pp. 327–340). Ursell (Proceedings of the London Mathematical Society, (2), vol. 37 (1934), pp. 402–415) has shown that the theorem is still true when hypothesis (2) is weakened to mere measurability. From one of his inequalities it follows that the total variation of $f(t+u)-f(t)$ is continuous in u for all u. By a theorem of Lusin we conclude that $f(t)$ is continuous. These two facts combined with Plessner's proof provide a short proof of Ursell's extension. (Received March 21, 1935.)

208. Mr. A. H. Fox: *Differential equations with continuous spectra.*

The differential equation $u''(x)+[l-V(x)]u(x)=0$ is studied by the use of the theory of transformations in Hilbert space. The case in which $V(x)$ is a real step-function with a finite number of steps gives certain results concerning the nature of the continuous and point spectra of the equation. A sequence of such equations is formed in which the step-functions converge to a continuous function with a finite number of maximum and minimum points. For this limiting equation the properties of the spectrum are determined by considering the manner in which the spectrum is affected by the limiting process. (Received March 23, 1935.)

209. Mr. Marshall Hall: *Prime divisors of second order sequences.*

Dr. Morgan Ward obtained a partial solution of the problem: given a second order recurrence and the initial terms of a sequence satisfying it, to determine whether or not a given prime will divide any term of the sequence (this Bulletin, vol. 40 (1934), pp. 825–828). This paper gives a complete solution of the problem, reducing the question, as did Ward, to the determination of the reduced period of a sequence. (Received March 20, 1935.)

210. Professor G. A. Hedlund: *A metrically transitive group defined by the modular group.*

It has been proved (Annals of Mathematics, 1934, pp. 787–808) that the geodesics on certain closed orientable surfaces obtained by identifying points congruent under certain Fuchsian groups constitute metrically transitive systems. The groups considered have a fundamental region lying interior to the unit circle. If the region under consideration is the upper half-plane and the group is the modular group, the existence of transitive geodesics is known (Artin and Herglotz). It can be shown that this system is metrically transitive. The method of proof is of necessity different from that in the cases previously considered due to the fact that the characterization of the geodesics by means of infinitive symbols is obtained in the present case by means of continued fractions. The proof in this case is somewhat simpler. (Received March 21, 1935.)

211. Mr. J. D. Hill: *Some theorems on double limits.*

In this note is considered the problem of determining, for an arbitrary function $f(x, y)$, whether or not the existence of the double limit $\lim_{x,y\to 0} f(x, y)$ is implied by the existence of a unique limit for f as (x, y) tends to $(0, 0)$ on every curve of a given class of curves passing through $(0, 0)$. A general criterion is formulated and by its application are obtained definite answers for five classes of curves; these classes include (a) the class of curves having analytic parametric representations, (b) the class of curves having parametric representations of class C^∞, and (c) the class of curves having continuously turning tangents at $(0, 0)$. (Received March 18, 1935.)

212. Professor E. V. Huntington: *Inter-relations among the four principal types of order.*

The four types of order whose inter-relations are considered in this paper are: (1) *serial order* (order of points on a directed straight line); (2) *betweenness* (order of points on an undirected straight line); (3) *cyclic order* (order of points on a directed closed line); and (4) *separation of pairs* (order of points on an undirected closed line). After recapitulating all the known sets of postulates which define each of these types as an abstract system the author establishes the following theorems. (A) Without restriction, 2, 3, and 4 may be defined in terms of 1; and 4 may be defined in terms of either 1 or 2 or 3. (B) With respect to an arbitrarily selected element of the given system, 1 and 2 may be defined in terms of 3; and 2 in terms of 4. (C) With respect to an arbitrarily selected pair of elements, 1 and 3 may be defined in terms of 2. (D) With

respect to an arbitrarily selected triad of elements, 1 and 3 may be defined in terms of 4. The paper will appear in the Transactions of the American Mathematical Society. (Received March 11, 1935.)

213. Professor E. V. Huntington: *The mathematical structure of Lewis's theory of strict implication.*

The formal postulates (or symbolic expressions) and the informal postulates (or rules of procedure) which occur in C. I. Lewis's theory of strict implication (Lewis and Langford, *Symbolic Logic*, 1932) are here rephrased in terms of a "subclass T," as an example of an abstract mathematical system. A comparison between "strict implication" and "material implication" is facilitated by numerical examples which illustrate the distinction between a "proposition" and the "assertion of a proposition"; and the connection between Lewis's system and Boolean algebra (cf. this Bulletin, vol. 40 (1934), pp. 729–735) is discussed. In particular, it is pointed out that a concept which may be called "effective implication" ($a=ab$) is definable in terms of "effective equality" and simple conjunction, without the use of negation or truth values. The paper will appear by invitation in the jubilee volume (vol. 25) of Fundamenta Mathematicae. (Received March 11, 1935.)

214. Professor R. L. Jeffery: *The inversion of approximate derivatives.*

Let $f(x)$ be a function which is finite almost everywhere on the linear interval (a, b). If there exists a sequence of summable functions $s_n(x)$ tending to $f(x)$ almost everywhere and such that $F(x)=\lim_{n\to\infty}\int_a^x s_n(x)dx$, then $f(x)$ is said to be integrable in the sequence sense to $F(x)$. The following results are proved: If $f(x)$ is finite almost everywhere on (a, b) and is almost everywhere equal to the approximate derivative of a continuous function $F(x)$, then $F(x)-F(a)=\int_a^x f(x)dx$, where the integration is in the sequence sense. Integration in the sequence sense includes the generalized Denjoy integral. The proofs of these results do not involve transfinite induction. Consequently integration in the sequence sense can be looked upon as a process of constructing the most general type of non-absolutely convergent integral without the use of transfinite numbers. (Received March 13, 1935.)

215. Professor Edward Kasner: *Characterization of the equilong group and a larger group.*

The only transformations which convert velocity families into velocity families form the conformal group. This characterization of conformal transformations was given by the author in 1906 (American Journal of Mathematics, 1906, p. 213). Later the same theorem was discovered by Scheffers, 1922. The present paper deals with the corresponding problem for line transformations preserving the dual-velocity type of doubly-infinite families. The solution is not the equilong group, as one might expect, but a larger group of the form $U=\phi(u)$, $V=v\chi(u)+\psi(u)$. This contains the equilong group as the special case $\chi=\phi'$. This type preserves the dual of the natural family type, studied in a paper read at the February meeting (abstract 41–3–131), obtained

in connection with certain infinitesimal contact transformations. (Received March 21, 1935.)

216. Professor B. O. Koopman: *On distributions admitting a sufficient statistic.*

It is shown in this paper that those univariate statistical distributions of analytical frequency function which admit of the application of R. A. Fisher's conception of a sufficient statistic have a very special exponential form. (Received March 15, 1935.)

217. Dr. E. R. Lorch: *Functions of self-adjoint transformations in Hilbert space.*

The subject of functions of self-adjoint transformations in an abstract Hilbert space has been treated principally by Haar, Neumann, F. Riesz, and Stone. We give a new approach to the general theory, developing for this purpose a theory of measure and integration for operators patterned upon that of Lebesgue for numerical functions. Starting with a resolution of the identity, we define in terms of closed linear manifolds the notion of measure of a linear set. Measurable functions are then introduced and the definition of the function of an operator follows immediately without introducing bilinear forms. The paper concludes with a demonstration of the principal properties of such functions of operators. (Received March 7, 1935.)

218. Professor J. A. Shohat: *The relation of the classical orthogonal polynomials to the polynomials of Appell.*

The fact that the derivatives of the classical orthogonal polynomials are also orthogonal polynomials (of the same kind) enables us, starting with such a polynomial of a given degree, to form a sequence of Appell polynomials of degree 0, 1, 2, $\cdots$. They are orthogonal in an extended sense (in the ordinary sense, only, in case of Hermite polynomials). Thus the theory of Appell polynomials becomes applicable, for example, to the discussion of the zeros of the polynomials in question, to the expansion of x^n in a polynomial series, etc. (Received March 21, 1935.)

219. Professor G. T. Whyburn: *Concerning sequences and limiting sets.*

A convergent sequence $[A_n]$ of closed sets in a compact metric space is said to converge regularly relative to r-cycles provided that for each $\epsilon>0$ there exist positive numbers δ and N such that if $n>N$ then any r-dimensional complete cycle in A_n of diameter $<\delta$ is ~ 0 in a subset of A_n of diameter $<\epsilon$. It is shown that regular convergence relative to 0-cycles for sequences of arcs and simple closed curves gives limiting sets of the same type. For sequences of topological spheres it gives cactoids as limiting sets, and in case the convergence is regular also relative to 1-cycles, these cactoids reduce to topological spheres. For a sequence of 2-cells whose boundaries also converge, regular convergence relative to 0-cycles gives as a limiting set a hemicactoid (in the sense of C. B. Morrey) whose base set is bounded by a boundary curve which is the

limit of the boundaries of the 2-cells. The hemicactoid reduces to its base set in case the convergence is regular also relative to 1-cycles, and this base set reduces to a 2-cell in case the boundaries of the 2-cells in the sequence converge regularly relative to 0-cycles. (Received March 23, 1935.)

220. Dr. S. L. Robinson: *Homogeneous sets which are disconnected by the removal of a finite number of points.*

The abstract spaces considered are not required to be Hausdorff topological spaces but only to satisfy the second property of Riesz. If the space P is totally disconnected and the group G of all homeomorphisms is transitive, G is k-fold transitive; if P is disconnected by the removal of one point and G is pseudo-3-fold transitive, G is pseudo-k-fold transitive for every k; while if P is disconnected by the removal of two points and G is pseudo-4-fold transitive, G is pseudo-k-fold transitive for every k and is doubly transitive. If G has pseudo-multiple transitivity of order k and of all lower orders and P is disconnected by the removal of $k-2$ points, then P is also disconnected by the removal of two points. If $k-2$ points are removed from P, the remaining space is connected if G is k-fold transitive and has less than k components if G is pseudo-k-fold transitive. (Received March 25, 1935.)

221. Dr. Saunders MacLane: *Abstract absolute values and polygonal irreducibility criteria.*

This paper investigates the connection between a certain type of irreducibility criterion for polynomials and the problem of determining all possible non-archimedean absolute values (*Bewertungen*) in a given polynomial ring. The irreducibility criteria concerned are the generalizations of Eisenstein's theorem which Dumas, Ore, and Kürschak have obtained by constructing Newtonian polygons corresponding to given polynomials. As Rella has pointed out (Journal für Mathematik, vol. 158), these generalizations depend on the use of an abstract absolute value for the coefficients in certain developments of the polynomials. The present paper shows that the same developments can be used to determine several successive absolute values in the corresponding polynomial ring. A suitable sequence of these absolute values will in turn give another type of absolute value for the ring, and these two types exhaust all the possible non-archimedean absolute values for the ring. The new absolute values thus obtained may be applied in turn to give new irreducibility criteria. The generalization consists essentially in going from the two moduli (a prime p and a polynomial $\phi(x)$, irreducible modulo p) used in the ordinary case, to a sequence of suitably chosen moduli, p, $\phi_1(x)$, $\phi_2(x)$, $\cdots$, $\phi_n(x)$. (Received March 25, 1935.)

222. Professor Hassler Whitney: *Sphere-spaces, with applications.*

To each point p of a complex K let there correspond an r-sphere $S_r(p)$ so that, roughly, the points on all spheres corresponding to a neighborhood U of p form the product of U and an r-sphere S_r. Then $S(K)$ is a *sphere-space*. Examples are the *tangent* and *normal* spaces to a manifold M_m in euclidean E_n,

formed by the unit spheres in the tangent or normal planes through points p of M, with p as center. Invariants are defined which show the structure in the large of $S(K)$: to each i-cycle Z (mod μ) corresponds a number $\gamma(Z)$ which is a topological invariant (mod μ); in certain cases, $\gamma(Z)$ is defined only mod 2. If the points of $S(K)$ form the product of K and S_r, these invariants all vanish. Conversely, the invariants characterize the space, at least for low dimensions. The well known invariants for vector distributions on a manifold M are the highest invariants of the tangent space. The invariants of the normal space may be defined as "local intersections"; they vanish in a number of (but not all) cases. In particular, the normal space to an orientable surface is a product space. (Received April 19, 1935.)

223. Dr. W. V. Quine: *A unified calculus of propositions, classes, and relations.*

Propositions (or truth-values), classes, and relations of all degrees are taken collectively as undiscriminated elements of the system; rather than being artificially segregated as usual by distinctive styles of variables and restricted ranges of operations, the above categories become distinguishable only through formal properties (as odd and even are distinguished within arithmetic). Operations traditionally restricted to propositions, to classes, to dyadic relations, etc., become so generalized as to apply to all elements. Simplification attends this generalization in that many traditional operations become cases of one unrestricted operation. Like Boolean algebra, the system avoids bound variables, types, and functions such as $\iota' x$, $x \epsilon \alpha$, etc., involving types; but it covers all else, e.g., truth-functions, Boolean functions, and the totality of relational functions such as converse, domain, relative product, perordinate, distributive referent, and their unfamiliar analogues for higher relational degrees. All are definable in terms of two binary operations, $\alpha * \beta$ and $\alpha \dagger \beta$, describable respectively as generalizations of the $\overline{\alpha \cup \beta}$ and $(\alpha \uparrow \beta \uparrow \alpha) \cap (\epsilon_a)_1$ of my *System of logistic*. Theorems issue from the formal postulates by two mechanical rules: (1) substitute for variables; (2) granted $X*(Y*Z)$ and $Y*(X*(W*W))$, put X for Y anywhere. (Received March 25, 1935.)

224. Professor D. V. Widder: *An application of Laguerre polynomials.*

By use of the Laguerre polynomials we give a new proof of the theorem of S. Bernstein to the effect that a function completely monotonic in $(0, \infty)$ can be represented by a Laplace-Stieltjes integral with non-decreasing determining function. The author obtains a new inversion formula for such an integral by a series of Laguerre polynomials. The method shows automatically that this series is summable in the sense of Abel to the determining function. It can be seen by other considerations that the series actually converges. (Received March 26, 1935.)

225. Dr. C. W. MacGregor: *The potential function method for the solution of two-dimensional stress problems.*

The potential function method for the determination of stresses and displacements in a body under plane stress or plane strain is developed further and

in a somewhat different manner than heretofore. The expressions for the stress components are derived in terms of potential functions and also in terms of functions $W(z)$ of a complex variable z for both the general two-dimensional problem and for special cases of loading on the semi-plane boundary. It is found that a large number of interesting solutions can be obtained in the case of the semi-plane from the complex function $W(z)=f(z)\cdot\log(z/c)$ where c is a unit of distance and $f(z)$ determines the distribution of loading along the boundary. The stress components are given explicitly for the cases where $f(z)=z^n$ in which the straight boundary of the semi-plane is acted upon either by shear or normal stresses varying as r^n. Expressions for the displacement components are also derived for both the general and special cases mentioned. (Received March 26, 1935.)

226. Dr. C. W. Vickery: *Sets of independent axioms for complete Moore space and complete metric space.*

A certain set of four axioms has been obtained. These axioms are shown to be independent and equivalent, in the topological sense, to axioms 0 and 1 of R. L. Moore's *Foundation of Point-Set Theory*. Axioms 0 and 2 of this set are the same as the corresponding axioms of Part IV (pp. 12–13) of the author's *Spaces in which there exist uncountable convergent sequence of points* (Tôhoku Mathematical Journal, vol. 40, part I, pp. 1–26); axiom 1 is a modification of axiom 1($\aleph_0$) of that paper; axiom 3 is a modification of axiom 4 of that paper. By substituting for axiom 1 a different modification of axiom 1($\aleph_0$) a set of independent axioms for complete space ($\mathfrak{D}$) of Fréchet is obtained. (Received March 28, 1935.)

227. Professor E. V. Huntington: *Postulates for effective equality and effective implication in formal logic.*

The purpose of this paper is to present an abstract mathematical theory $(K, T, \times, \doteq)$ which is capable of being interpreted as that part of the calculus of propositions which is expressible in terms of conjunction and equality alone, without the use of negation or truth-values. A set of independent postulates for "$a\times b$," "$a\doteq b$," and "a in T" is given, and a relation called "effective implication" is defined by $(a<b)=(a\doteq ab)$. This relation proves to resemble "strict implication" more closely than "material implication." The "general rules of procedure" usually implicit in the proofs of theorems are listed explicitly. The paper will appear in the Proceedings of the National Academy of Sciences. (Received March 29, 1935.)

228. Professor H. A. Simmons and Mr. William Block: *Classes of maximum numbers associated with certain symmetric equations in n reciprocals.*

As in two previous papers of Simmons (Transactions of this Society, vol. 34, pp. 876–907) and Stelford and Simmons (this Bulletin, vol. 40, no. 12), we let $\Sigma_{\lambda,\mu}(x)$ stand for the elementary symmetric function of the μth order of the variables x_i $(i=1, \cdots, \lambda)$. The equations relative to which the above authors have identified classes of maximum numbers are of the forms (1) $\Sigma_{n,r}(1/x)$

$+\lambda\Sigma_{n,r+1}(1/x)=b/a$, $a\equiv(c+1)b-1$, $n>r$; (2) $\Sigma_{i=r}^{s}\Sigma_{n,i}(1/x)=b/a$, $n\geqq s>r$. In the present paper we find that by means of numerous modifications and extensions of methods which were employed in the two articles mentioned above we are able to identify (our usual) classes of maximum numbers relative to the equation (3) $\Sigma_{i=r}^{s}\lambda_i\Sigma_{n,i}(1/x)=b/a$, $n\geqq s>r$, where the λ's are such that (3) includes both (1) and (2) as (rather) special cases. If $(x_1\cdots x_n)^{-1}$ be added to the left member of (3) and $n\geqq s+2$, the new equation with coefficients as we restrict them to be for (3) is still amenable to our method of attack. (Received March 11, 1935.)

229. Professor H. R. Brahana: *Metabelian groups and trilinear forms.*

The metabelian group $G=\{H, U_1, U_2, \cdots, U_m\}$, with central of order p^{n-k} and commutator subgroup of order p^l, which is a subgroup of the holomorph of the abelian group H of order p^n and type 1, 1, $\cdots$ determines a trilinear form $F(x, y, z)=\Sigma a_{hij}x_hy_iz_j$, $h=1, 2, \cdots, k$; $i=1, 2, \cdots, l$; $j=1, 2, \cdots, m$; a_{hij} in a $GF(p)$. Conversely, a form F determines a group G. A necessary and sufficient condition that G and G' be simply isomorphic is that F and F' be conjugate under linear transformations in $GF(p)$ on the x's, the y's, and the z's. Modular invariants of F are immediately expressible in terms of properties of G. In the definition of F by means of G the variables x, y, and z have special roles; in the classification of the forms F they are indistinguishable. Hence, a set of properties which determines G determines F which in turn determines two more groups G' and G''. The three groups G, G', and G'' are in general distinct. Thus the work of classification of the groups G is reduced to approximately one-third. (Received March 18, 1935.)

230. Professor I. A. Barnett, Dr. C. W. Mendel, Mr. Haim Reingold: *Generalized determinants of Vandermonde.*

This paper is concerned with determinants whose elements are the elements of different powers of a square matrix. Let G_{ij}, $i, j=1, 2, \cdots, n$, be a square matrix the determinant of which is denoted by g; let $G_{rs}^{(m)}$ stand for the element in the rth row and sth column of the mth power of the matrix G_{ij}, and let $G_{ij}^{(c)}$ denote the Kronecker δ. Consider the $n\times n$ determinant $\left|G_{r_1s_1}^{(m_1)}G_{r_2s_2}^{(m_2)}\cdots G_{r_ns_n}^{(m_n)}\right|$, $i=1, 2, \cdots, n$, in which the ith row is exhibited, where $r_1, r_2, \cdots, r_n$ and $s_1, s_2, \cdots, s_n$ are arbitrary permutations of the integers $1, 2, \cdots, n$, and $m_1<m_2<\cdots<m_n$; we denote this determinant by $\Delta(m_1, m_2, \cdots, m_n)$. The following results are proved: (1) $\Delta(1, 2, \cdots, n)=g\Delta(0, 1, \cdots, n-1)$; (2) $\Delta(k+1, k+2, \cdots, k+n)=g^{k+1}\Delta(0, 1, \cdots, n-1)$; (3) $\Delta(m_1, m_2, \cdots, m_n)=f(a_0, a_1, \cdots, a_n)g^{m_1}\Delta(0, 1, \cdots, n-1)$, where f is a rational integral function of the coefficients of the characteristic equation of the matrix G_{ij}. For the particular square matrix $G_{ij}=\Sigma_{i=1}^{n}x_i\delta_{ij}=x_j$ the determinant $\Delta(0, 1, \cdots, n-1)$ reduces to the determinant of Vandermonde. (Received March 21, 1935.)

231. Dr. E. D. Jenkins: *On the composition of quadratic forms.*

The problem considered is to find a computational method for determining a compound of binary quadratic forms. Dedekind established a correspondence

between the composition of forms and the multiplication of moduls of an algebraic field. Chatelet and MacDuffee have shown that the theory of matrices is very effective in the treatment of algebraic moduls and ideals. It is possible to define a class of integral algebraic moduls, not as general as those considered by Dedekind, but such that the existing theorems concerning the correspondence between ideals and matrices with rational integral elements will carry over to this particular type of modul. The result is a matric method for the composition of forms. The characteristics which serve to recommend this method are first, its simplicity, and, second, the saving of time involved in its use for computational purposes. (Received March 20, 1935.)

232. Dr. W. T. Reid: *A boundary value problem associated with the calculus of variations.* II.

The author has treated a boundary problem linear in a parameter, and associated with the problem of Bolza in the calculus of variations (American Journal of Mathematics, vol. 54 (1932), pp. 769–790). The purpose of the present paper is two-fold: (1) to indicate for the theorems of the previous paper concerning the existence of characteristic numbers a proof which is simpler in detail and more closely related to classical methods of the calculus of variations; (2) to treat a related boundary problem which is in general non-linear in the parameter. The latter boundary problem is assumed to satisfy hypotheses similar to those used by Morse in a recent treatment of a self-adjoint second-order differential system (American Mathematical Society Colloquium Publications, vol. 18, chapter IV), but the method used here differs from that of Morse. There is associated with the given boundary problem an auxiliary problem which is linear in a second parameter, and the characteristic numbers of the associated linear problem are considered as functions of the original parameter. Comparison and oscillation theorems for the given problem are consequences of corresponding theorems for the associated problem, and for this latter problem these theorems follow from the extremizing properties of the characteristic numbers. (Received March 22, 1935.)

233. Dr. M. S. Robertson (National Research Fellow): *Analytic functions star-like in one direction.*

Analytic functions $f(z)=z+\sum_2^\infty a_n z^n$ are considered which are regular for $|z|<1$ and which map each circle $|z|=r<1$, for values of r near 1, on contours C_r having the property that there exists at least one straight line through the origin which cuts each C_r in not more than two points. Then $|a_n|\leqq n^2$ for all n and equality is attained for essentially only one function of the class. If $f(z)$ is odd then $|a_n|\leqq n$. If the straight line in question is the real axis and if the coefficients are real, then $|a_n|\leqq n$ and $f(z)$ is then typically-real. Further, $\lim_{r\to 1} f(re^{i\theta})$ exists as a finite limit for almost all θ. If $g(z)$ is univalent for $|z|<1$, $g'(0)=1$, and maps $|z|=r<1$, for values of r near 1, on a contour convex in one direction (coefficients complex and denoted by c_n) then $|c_n|\leqq n$ for all n and equality is attained for a fixed n for only one function of this class. This constitutes a proof of the Bieberbach conjecture for this class of univalent functions. (Received March 15, 1935.)

234. Professor Wayne Dancer: *Concerning symmetrical cut-sets.*

The notion of symmetrical cut-point (=*s. c-p.*) of a connected point set, introduced by Gehman, may be extended in an obvious manner to the notion of symmetrical cut-set (=*s. c-s.*). Gehman showed how to characterize the simple arc in terms of *s. c-p.* In the present paper it is shown, for instance, that the simple closed curve and surface may be characterized in terms of *s. c-s.* Thus, the simple closed curve is a connected and locally connected space of which every pair of distinct points is an *s. c-s.* The strong *s. c-s.* is defined to be one which, under the transformation defining the notion of *s. c-s.*, remains point-wise invariant. It is shown that the only strong *s. c-s.* of the simple closed surface is the simple closed curve, and that the simple closed surface may be characterized as a Jordan continuum which is not disconnected by any pair of points, and of which every simple closed curve is a strong *s. c-s.* (Received March 22, 1935.)

235. Professors Otto Laporte and G. Y. Rainich: *Pseudominimal hypersurfaces and harmonic functions.*

This paper develops the subject studied in an earlier paper (see abstract 41–1–6) generalized to any number of dimensions. The integral representation of a *general* hypersurface is replaced by a representation in terms of a potential-like function ϕ and its derivatives and involving the same stereographic parameters used before. Due to the restricted transformation group a simplified form of tensor analysis results. Any conditions imposed on the curvature may be expressed as a differential equation on ϕ. For *pseudominimal* hypersurfaces this becomes a linear differential equation of second order. By a method analogous to that of separation of variables in mathematical physics an infinity of particular solutions is obtained which is of the form: "solid" harmonic times hypergeometric function of the radius. This establishes between pseudominimal hypersurfaces and harmonic functions a relationship generalizing that between minimal surfaces and analytic functions. In spaces of even dimensions this relationship can be given a closed form which, for example, for four dimensions becomes: $\phi=6(1-r^2)H+6(1+r^2)x_\rho\partial H/\partial x_\rho+(1+r^2)^2x_\rho x_\sigma\partial^2 H/\partial x_\rho\partial x_\sigma$. (Received March 22, 1935.)

236. Mr. C. B. Tompkins: *Linear connections in normal space.*

The author considers the formulas which express the derivatives of the coordinate vectors of an m-dimensional variety in an n-dimensional euclidean space. At each point of the variety there exists an $(n-m)$-dimensional normal space in which is chosen a set of mutually orthogonal unit vectors. The derivatives of the coordinate vectors and of these vectors are expressed as linear combinations of the set of vectors. Directions of reference in the normal space are not fixed. The normal components of the derivatives of the normal vectors are given by antisymmetric coefficients N_k^{pq} somewhat similar to the Christoffel symbols; they may be used as connection coefficients. From them the analogue of the Riemann tensor, S_{ik}^{pq}, is constructed and it is expressed in terms of the analogues of the coefficients of the second differential form of surfaces.

The vanishing of this tensor implies that there exists in the tangent space a set of mutually orthogonal vectors along which lie all the determinate principal directions of the indicating quadrics of the variety. Curves lying along the principal directions of these quadrics possess many of the properties of lines of curvature. (Received March 22, 1935.)

237. Mr. L. R. Wilcox: *Pairs of surfaces in five-dimensional space.*

The purpose of this paper is to construct a projective differential geometry of a pair of analytic surfaces immersed in a linear space of five dimensions, with an analytic one-to-one point correspondence between them. The theory is based on a system of partial differential equations and a set of power series expansions representing the surfaces in a neighborhood of a pair of corresponding points. Several families of curves on the surfaces and a number of covariant configurations associated with the surfaces are defined, and projective interrelations among them are studied. The analytic basis for a theory of envelopes and loci of the covariant configurations is constructed, and a portion of this theory is developed. (Received March 18, 1935.)

238. Professor A. A. Albert: *Normal division algebras of degree p^e over F of characteristic p.*

The author recently proved that if A is a normal division algebra of degree p over a field F of characteristic not p then A is cyclic if and only if A contains a sub-field $F(y)$, $y^p = g$ in F. In this paper the author considers algebras over F of characteristic p and proves that A of degree $p^e = n$ over F is cyclic if and only if A has a maximal sub-field $F(y)$, $y^n = g$ in F. (Received March 22, 1935.)

239. Professor C. C. MacDuffee: *Covariants of r-parameter groups.*

A covariant (contravariant) of an r-parameter group is defined as an invariant of the group and its first (second) parameter group. A general replacement theorem is proved showing that every absolute or relative covariant is a function of certain elementary covariants. It is shown that any set of equations defining an invariantive property can be put into covariant form. It is shown how the principal theorems of both algebraic and differential covariant theory appear as special instances. From this standpoint the tensor analysis is plainly a too narrow formulation of the covariant theory of differential forms. (Received March 20, 1935.)

240. Professor G. A. Bliss: *Abnormality in the calculus of variations.*

For a problem of Bolza in the calculus of variations Graves recently proved an analogue of the necessary condition of Weierstrass, and Hestenes generalized the Mayer condition and made an ingenious sufficiency proof, for an arc without normality restrictions. In the present paper the author analyzes the significance of abnormality. The papers of Graves and Hestenes for the first time bring to a satisfactory close an important chapter in the theory of arcs

which are normal but not necessarily normal on sub-intervals. The abnormal accessory minimum problems recently studied by Carathéodory can be reduced to normal ones, but in general an abnormal minimizing arc is analogous to a singular point of implicit functions, and a complete theory of such arcs is likely to be difficult and complicated. The sufficient conditions described by Hestenes are not in general necessary, but they are the first ones established for such arcs. A statement in a recent paper by Morse might easily be interpreted to the contrary. In this paper of Morse an interesting lemma, first announced by Reid but earlier proved by Morse, is applied to extend to abnormal cases the effectiveness of a theorem of Bliss with the help of the results of Hestenes. (Received March 21, 1935.)

241. Professor A. A. Albert: *On the Hilbert irreducibility theorem.*

A field F is called an H.I. field if the Hilbert irreducibility theorem holds in F. W. Franz proved that every separable algebraic extension K of finite degree over an H.I. field F is an H.I. field, but was unable to treat the case where K is inseparable over F. The author now proves that every algebraic extension K of finite degree over an H.I. field F is an H.I. field. (Received March 22, 1935.)

242. Professor A. A. Albert: *Simple Lie algebras over a non-modular field.*

A Lie algebra L over a non-modular field F will be called normal simple over F if H is an algebraically closed extension of F and L_H is a simple algebra. In this paper the author proves that the adjoint enveloping associative algebra A of L is a total matric algebra of degree s over a field K of degree t over F and that the order of L over F is $n=st$. Moreover the basal units of L are linearly dependent in K and L is a normal simple Lie algebra of order s over K with A over K as adjoint enveloping algebra. (Received March 22, 1935.)

243. Mr. E. B. Escott: *Amicable numbers.*

This paper is an extension of a paper with the same title read at a meeting of the Chicago section on April 19, 1930. By means of extensions of Euler's methods, the author has been able to extend the number of amicable pairs of numbers to over 360 pairs. The paper will be published in Scripta Mathematicae. (Received March 22, 1935.)

244. Dr. Gertrude S. Ketchum: *On certain generalizations of the Cauchy-Taylor expansion theory.*

Okada and others have established the existence of expansions for functions regular at the origin in a set of functions $F_n(x)=x^n+\sum_{s=1}^{\infty} a_s^{(n)}x^{n+s}$ where $F_n(x)$ is analytic and is bounded with respect to n in some region about zero. We extend the region of convergence previously obtained and in important classes of such expansions establish the maximum region of convergence. A class of sets of integral functions is defined for which the expansion of functions analytic for $|x|<\rho$ is valid in the region $|x|<\rho$. The Bessel coefficients belong to

this class. We also develop a very general theorem on expansions in products of functions of the form $F_n(x)$. When restricted to Bessel functions this theorem gives expansions of the form $x^\kappa f(x)=\sum_{n=0}^{\infty} c_n J_{\sigma_n^{(1)}}(x) J_{\sigma_n^{(2)}}(x) \cdots J_{\sigma_n^{(k)}}(x)$ where σ is restricted to an unbounded region of the complex plane such that $n/|\sigma+n|$ is bounded, $\kappa+n=\sigma_n^{(1)}+\sigma^{(2)}+\cdots+\sigma_n^{(k)}$, and $\lim_{n\to\infty}|\sigma_n^{(r)}|=\infty$, $r=1, 2, 3, \cdots, k$. (Received March 20, 1935.)

245. Dr. P. W. Ketchum: *Expansions of functions holomorphic in distinct regions.*

Let $f(x)$ be any function holomorphic in each of a set of non-overlapping regions R_k, $(k=1, 2, \cdots, q)$, and let a_k be any point interior to R_k. Let $F_{n,m}(x)$, $(n=0, 1, \cdots; m=1, 2, \cdots, q)$, have a zero of order n at a_k if $m=k$ and of order higher than n if $m\neq k$. Suppose a function $\theta_k(x)$ exists such that (1) $\theta_k(x)$ has a simple zero at a_k, (2) $\lim_{x\to a_k} F_{n,k}(x)[\theta_k(x)]^{-n}=1$, and (3) $|F_{n,m}(x)[\theta_k(x)]^{-n}|$ is bounded in R_k by a constant M independent of n, m, and k. Then there exists a unique set of numbers $\alpha_{n,m}$ such that $\Sigma_{n,m}\alpha_{n,m}F_{n,m}(x)$ converges absolutely and uniformly to $f(x)$ in each of a set of regions R_k', $k=1, 2, \cdots, q$, where R_k' contains a_k in its interior. (Received March 21, 1935.)

246. Professor W. J. Trjitzinsky: *Linear difference equations containing a parameter.*

The equation under consideration is of order n. Its coefficients are analytic functions of x and of a parameter λ, asymptotic in these variables to series in negative integral powers of x and of λ, with possibly a few positive powers of x and of λ present. The emphasis of the paper being on the investigation of the asymptotic properties of solutions in so far as the complex parameter λ is concerned, the equation is assumed to be formally of Fuchsian type in x. On the other hand, no further restrictions are made with regard to the parameter. The main result is embodied in the existence theorem, according to which there always exists a fundamental set of solutions asymptotic in x and in λ to certain formal series. (Received March 21, 1935.)

247. Dr. E. W. Miller: *Concerning polar singularities of an analytic function.*

Consider the analytic function $f(z)$ defined by the power series $\Sigma a_n z^n$ whose circle of convergence, C, has a radius 1. It is shown that $f(z)$ has $z=1$ as a pole of order m and has no further singularity on C if and only if there exists a polynomial $g(x)$ of degree $m-1$ such that $\overline{\lim}_{n=\infty}|A_n-A_{n+1}|^{1/n}<1$, where $A_n=a_n/g(n)$. It is also shown that $f(z)$ has a pole of order m and no further singularity on C if and only if there exists a polynomial $g(x)$ of degree $m-1$ such that $\overline{\lim}_{n=\infty}|A_n^2-A_{n-1}A_{n+1}|^{1/n}<1$, where $A_n=a_n/g(n)$. These theorems are simple generalizations of well known theorems of Pringsheim and Hadamard. A few applications are made and a theorem on limits which is closely connected with these matters is proved. (Received March 25, 1935.)

248. Dr. E. W. Miller: *On certain properties of Fréchet L-spaces.*

In different articles in Fundamenta Mathematicae W. Sierpinski, C. Kuratowski, and B. Dushnik have considered relations among the following properties in L-space: (A) the closure of every set is closed; (B) every nondenumerable set contains an element of condensation; (C) every well-ordered series of decreasing closed sets is denumerable; (D) every well-ordered series of increasing closed sets is denumerable; (E) every set Q contains a denumerable set D such that $Q \subset D+D'$. Dushnik raises the following questions. (1) Do conditions B and D together imply E? (2) Do conditions C and E together imply B? In the present note two L-spaces are constructed which show that these questions are to be answered in the negative. That the spaces in question fulfill the desired requirements appears as a consequence of two theorems which are proved for the linear continuum and which are concerned with properties somewhat stronger than C and D. (Received March 25, 1935.)

249. Professor H. P. Thielman: *Note on the factorization o Fredholm kernels.*

This paper establishes the theorem: if $G(x, y)$ is a real, positive, definite, symmetric kernel, there exist an infinite number of real, positive, definite, symmetric kernels $K(x, y)$ such that $G(x, y)=K(x, y)+K(y, x)+\int_a^b K(x, t)K(t, y)dt$, where $G(x, y)$ and $K(x, y)$ are such that their squares are integrable over $a \leqq x \leqq b$, $a \leqq y \leqq b$. This theorem is an analogue of a well known theorem on finite matrices (see C. C. MacDuffee, *The Theory of Matrices*, 1933). The result is used to prove that every Riemannian function space of a certain type can be immersed in a euclidean function space. (Received March 27, 1935.)

250. Professor Einar Hille: *Notes on linear transformations.* I.

We consider linear transformations of form $K_\alpha[f]=\alpha\int_{-\infty}^{+\infty} K(\alpha u)f(u+x)du$, in particular the following problems: (i) solutions of $K_\alpha[f]=0$, (ii) solutions of $K_\alpha[f]=f$, (iii) functional equations satisfied by $K_\alpha[K_\beta[f]]$, (iv) metric properties, including properties of contraction and degree of approximation of f by $K_\alpha[f]$. The analysis is carried out in detail for the kernels associated with the names of Dirichlet, Picard, Poisson, and Wierstrass. (Received April 2, 1935.)

251. Professor W. J. Trjitzinsky: *Linear functionals in the theory of quasi-analytic and D-analytic classes.*

Applications are made of the properties of linear functionals, relating to certain metric function spaces, to the theory of quasi-analytic and D-analytic classes. These developments are along lines similar to those of the author's recent work on quasi-analytic functions (Mathematische Zeitschrift, (1935), pp. 560–590). (Received April 16, 1935.)

252. Professor Oscar Zariski: *A topological proof of the Riemann-Roch theorem.*

The proof is based upon the consideration of the symmetric n-th product

V_n of a Riemann surface of genus p. Fundamental for the proof is the expression of a linear series g_n^r on an algebraic curve of genus p as a cycle Γ_{2r} on V_n in terms of the cycles of a base for weak homologies on V_n. Recourse to algebra or to function theory is confined to the following elementary and least significant part of the Riemann-Roch theorem: if the g_n^r is complete, then $r \geqq n-p$. The remainder of the theorem ($r=n-p$, if $n>2p-2$; the uniqueness of the canonical series g_{2p-2}^{p-1}; the formula $r=n-p+i$ for series of index of speciality i) is proved topologically by means of the intersection properties of the cycles on V_n. (Received April 15, 1935.)

253. Mr. J. F. Wardwell: *Continuous transformations preserving all topological properties.*

If A and B are any compact metric spaces and $T(A)=B$ is a continuous transformation, let G_0 denote the collection of all non-degenerate sets of the collection $[T^{-1}(b)]$, for all points b of B. Let G_i denote the collection of all sets of G_{i-1} which intersect L_{i-1} = limit superior of G_{i-1}, for $i=1, 2, 3, \cdots$. If the following conditions are satisfied: (1) for any $\epsilon>0$, any set g of G_0, and any point x of g, there exists a homeomorphism $W(A-x)=A-g$ which is the identity outside of the ϵ-neighborhood of g in A, (2) there exists some number α of the first or second number class such that $G_\alpha=0$, and (3) $\Pi_1^\infty L_i$ is a zero-dimensional set, it is shown that there exists a topological transformation $S(A)=B$. (Received April 19, 1935.)

254. Professor Ervand Kogbetliantz: *On the jump of a function determined by its Hermite or Laguerre series.*

Using some previous results concerning the asymptotic behavior of the "singular" Hermite or Laguerre series, the author gives in the present paper a complete treatment of Gibbs' phenomenon for Hermite and Laguerre series. The situation turns out to be quite different from that familiar in the theory of trigonometric Fourier series. (Received April 26, 1935.)

255. Professor Wilhelm Maier: *Addition theorem of Euler's* Γ*-function.*

Hölder's classical statement concerning the non-existence of differential equations of finite order and algebraic coefficients satisfied by $\Gamma(u)$ suggests the construction of transcendent identities, comparable to the algebraic addition theorems of doubly periodic functions. Generalizing Newton's binomial theorem for fractional marks of summation $\nu \equiv x \pmod 1$, we find for certain values of a, b, λ the convergent expansion:

$$(a+b)^\lambda/\Gamma(1+\lambda)=\sum_{\pm\infty}^{\nu\equiv x(1)} (a^\nu b^{\lambda-\nu})/[\Gamma(1+\nu)\Gamma(1+\lambda-\nu)].$$

If besides convergence of the above, two initial conditions for $\Gamma(1+\nu)$ are granted, we are led to a new characterization of $\Gamma(u)$. (Received April 9, 1935.)

256. Mr. H. F. S. Jonah: *A functional equation of certain Vandiver polynomials.*

By means of a study of the generating function of the Bernoulli polyno-

mials, the author has obtained a quadratic functional equation of a function closely related to the $h_n^{(k,l)}(x)$ of Vandiver (Annals of Mathematics, vol. 27 (1926), pp. 171–176). The functions used in this work are slightly more general than Vandiver's but can be expressed in terms of the $h_n^{(k,l)}(x)$. Using this functional equation we are able to obtain a transformation of the Kummer criteria, in particular, by an appropriate choice of our parameters, we are able to transform the congruences given by Mirimanoff (Journal für Mathematik, vol. 128 (1903), pp. 45–48); also the congruences given by Vandiver (Proceedings of the International Congress at Toronto, (1924), p. 679); also the congruences given by Vandiver (this Bulletin, vol. 28 (1922), p. 258); finally, the more general congruences given by Vandiver (Annals of Mathematics, vol. 27 (1926), pp. 171–176.) (Received April 9, 1935.)

257. Mr. Henry Scheffé: *Asymptotic solutions of certain linear differential equations in which the coefficient of the parameter may have a zero.* Preliminary report.

Asymptotic solutions for large values of the parameter ρ of the equation $u^{(n)}(z,\rho)+0+p_2(z)u^{(n-2)}(z,\rho)+\cdots+\{p_n(z)-\rho^n\phi^n(z)\}u(z,\rho)=0$, where $p_i(z)$ and $\phi^n(z)$ are analytic, may be found from Birkhoff's paper (Transactions of this Society, vol. 9 (1908), p. 219) for z on a finite interval on the real axis on which $\phi^n(z)\neq 0$. In this paper are obtained forms valid when z is permitted to range over a finite region of the complex plane in which $\phi^n(z)$ has a zero of order ν. The second order case with lighter restrictions has been treated by Langer (Transactions of this Society, vol. 34 (1932), p. 447). As in his case, the forms depend primarily on an auxiliary variable $\xi=\rho\int_0^z\phi(z)dz$. The method, suggested by Langer, yields for each of certain configurations of z and ρ, and when $|\xi|>N$, formulas for n solutions of independent asymptotic form. Also, two sets of formulas are found for the principal solutions at the origin, for $|\xi|>N$ and $|\xi|<N$, respectively. All the results are explicit to factors $\{1+O(\xi^{-1})+O(\rho^{-\alpha})\}$ for $|\xi|>N$, and $\{\sigma(\xi)+O(\rho^{-\alpha})\}$ for $|\xi|<N$, where $\alpha=n/(n+\nu)$, and $\sigma(\xi)$ is a known power series. (Received April 18, 1935.)

258. Professor L. E. Dickson: *Cyclotomy when e is composite.*

This paper will appear in the Transactions of this Society, as a sequel to a paper in its current volume. There the author treated the case when e is a prime or double a prime. In this paper the author treats new cases when e is composite, and meets new difficulties. (Received April 19, 1935.)

259. Professor G. T. Whyburn: *Regular convergence and monotone transformations.*

In this paper it is first shown that if, in a compact metric space S, the sequence of closed sets $[A_n]$ converges to the limiting set A regularly relative to k-cycles (see the abstract of the author's paper *Concerning sequences and limiting sets*, No. 41–5–219), for every $k\leq r$, then A is locally γ^r-connected. A continuous transformation $T(A)=B$ is said to be r-monotone provided that for each $b\in B$ and each $k\leq r$, the kth connectivity number of $T^{-1}(b)$ is zero.

Let the sequence of r-monotone transformations $T_n(A)=B_n$ converge uniformly to the limit transformation $T(A)=B$, where A, B, and B_n are closed and contained in S. Then in order that B_n converge to B regularly relative to r-cycles it is necessary and sufficient that B be locally γ^r-connected for every $k \leqq r$ and that T be r-monotone. (Received May 3, 1935.)

260. Mr. I. N. Kagno: *Irreducible non-projective planar graphs.* Preliminary report.

The author's methods of studying and classifying irreducible non-toral graphs are applied to irreducible graphs which cannot be mapped on a projective plane. It is shown that an irreducible non-projective planar graph has at least seven and at most fourteen vertices. The existence of irreducible graphs which cannot be mapped on a non-orientable surface of any given genus is established. (Received April 30, 1935.)

261. Professor T. Y. Thomas: *Algebraic characterizations in complex differential geometry.*

In this paper the idea of the algebraic characterization is introduced and the characterizations of the metric representations of affinely connected spaces are considered. The methods used are quite general and permit a wide range of applications. (Received April 30, 1935.)

262. Professor J. W. Campbell: *On the principles of Hamilton and Cartan.*

A holonomic dynamical system can be characterized by Hamilton's stationary action or by a Cartan integral invariant, and recently A. E. Taylor has extended Cartan's principle to non-holonomic systems. In this paper an equivalence theorem is proved and is then used to show more clearly the correspondence between the principles of Hamilton and Cartan. Both holonomic and non-holonomic systems are discussed; in the non-holonomic case the form of Cartan's integral invariant given by Taylor is obtained by a different and simpler method and the corresponding stationary integral of Hamilton is also obtained. From the latter there results an extension of Hamilton's principle to non-holonomic systems in the ordinary sense, viz. $\delta\int_{t_0}^{t_1} L dt=0$, when comparison is made, with fixed end points, between the path followed and neighbouring varied paths which are kinematically possible. (Received May 1, 1935.)

263. Mr. Garrett Birkhoff: *On the combination of topologies.*

The notion of the relative strength of different topologies over a fixed set of points is well known. One can, however, obtain a sharper characterization of relative topologies by introducing two binary operations ("meet" and "join"). The existence and comparison of these operations with respect to the primitive ideas of distance, neighborhood, convergence, and closure occupies most of the paper. (Received May 6, 1935.)

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

ANTWEILER (A.). Unendlich. Eine Untersuchung zur metaphysischen Wesenheit Gottes auf Grund der Mathematik, Philosophie Theologie. (Freiburger theologische Studien, Heft 38.) Freiburg i. Br., Herder, 1934. 200 pp.

BARAVALLE (H.). Zahlen für Jedermann. 5te veränderte Auflage. Stuttgart, Franckh'sche Verlagsbuchhandlung, 1934. 166 pp.

COLERUS (E.). Vom Einmaleins zum Integral. (Mathematik für Jedermann.) Berlin, Paul Zsolnay Verlag, 1934. 404 pp.

COURANT (R.). Differential and integral calculus. Volume 1. Glasgow, Blackie, 1934. 562 pp.

EMDE (F.). See JAHNKE (E.).

ENRIQUES (F.). Signification de l'histoire de la pensée scientifique. (Actualités Scientifiques et Industrielles, No. 161: Philosophie et histoire de la pensée scientifique, Exposés publiés sous la direction de F. Enriques, I.) Paris, Hermann, 1934. 68 pp.

FRITZ (B.). Theorie und Berechnung vollwandiger Bogenträger bei Berücksichtigung des Einflusses des Systemverformung. Berlin, Springer, 1934. 142 pp.

JAHNKE (E.) and EMDE (F.). Funktionentafeln. Leipzig, Teubner, 1934. 330 pp.

LIETZMANN (W.). Altes und Neues vom Kreis. (Mathematisch-physikalische Bibliothek, Reihe 1, Band 87.) Leipzig und Berlin, Teubner, 1935. 4+47 pp.

MAHAJANI (G. S.). Lessons in elementary analysis. 2d edition. Poona, Aryabhushan Press, 1934. 264 pp.

QUINE (W.). A system of logistic. Cambridge, Harvard University Press, and London, Oxford University Press, 1934. 11+204 pp.

SMITH (D. E.). The poetry of mathematics and other essays. (The Scripta Mathematical Library, No. 1.) New York, Scripta Mathematicae, 1934. 6+91 pp.

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ZARISKI (O.). Algebraic surfaces. (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 3, Heft 5.) Berlin, Springer, 1935. 198 pp.

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BAILEY (A. A.). The consciousness of the atom. A series of lectures delivered in New York City. London, Watkins, 1934. 163 pp.

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BENEDICKS (C.). Nouveaux résultats expérimentaux sur l'effet électrother-

mique homogène. (Actualités Scientifiques et Industrielles, No. 130.) Paris, Hermann, 1934.

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BOUASSE (H.). Optique et photométrie dites géométriques. Paris, Bibliothèque Scientifique de l'Ingénieur et du Physicien, 1934. 31+620 pp.

CURIE (M.). Luminescence des corps solides. (Recueil des Conférences-Rapports de Documentation sur la Physique, Volume 24.) Paris, Les Presses Universitaires de France, 1934. 3+140 pp.

DUBROCA (M.). La masse et l'énergie. Le noeud gordien de la doctrine einsteinienne, Esquisse d'une méthode rationnelle en physique. Paris, Gauthier-Villars, 1934.

EVE (A. S.). Physics. (Home University Library, No. 173.) London, Butterworth, 1934. 258 pp.

FELDMAN (W. M.). Biomathematics: being the principles of mathematics for students of biological science. 2d edition. London, Griffin, 1935. 18+480 pp.

FISHER (R. A.). Statistical methods for research workers. (Biological Monographs and Manuals.) 5th edition. Edinburgh and London, Oliver and Boyd, 1934. 14+319 pp.

GENARD (J.). Fluorescence des vapeurs dans le champ magnétique (spectres de fluorescence moléculaire). (Actualités Scientifiques et Industrielles, No. 177.) Paris, Hermann, 1934. 47 pp.

HAISSINSKY (M.). Les radiocolloides. (Actualités Scientifiques et Industrielles, No. 185.) Paris, Hermann, 1934. 1+25 pp.

HAUSHOFER (K.). Raumüberwindende Mächte. (Macht und Erde, Band 3.) Leipzig und Berlin, Teubner, 1934. 7+359 pp.

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HULVEY (C. N.). The mathematics of finance. New York, Macmillan, 1934. 12+306 pp.

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HUYGENS (C.). Oeuvres complètes. Publiés par la Société Hollandaise des Sciences. Tome 18: L'Horloge à pendule ou à balancier de 1666 a 1695; anecdota. La Haye, Nijhoff, 1934. 4+703 pp.

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JACKSON (L. C.). Low temperature physics. (Methuen's Monographs on Physical Subjects.) London, Methuen, 1934. 7+122 pp.

JACOB (L.). High voltage physics. (Methuen's Monographs on Physical Subjects.) London, Methuen, 1934. 7+107 pp.

JAMESON (A. H.). Advanced surveying; a textbook for students. London, Pitman, 1934. 7+360 pp.

KEMP (P.). Theory of alternating current wave forms. (A Series of Monographs on Electrical Engineering, I.) London, Chapman and Hall, 1934. 9+218 pp.

LIWSCHITZ (M.). Die elektrischen Maschinen. Band 3. Leipzig und Berlin, Teubner, 1934. 409 pp.

MILLIKAN (R. A.). Electrons (+ and −), protons, neutrons and cosmic rays. Chicago, University of Chicago Press, 1934. 187 pp.

PERRIN (J.). Grains de matière et de lumière. Partie 1: Existence des grains. Partie 2: Structure des atomes. Partie 3: Noyaux des atomes. Partie 4: Transmutations provoquées. (Actualités Scientifiques et Industrielles, Nos. 190, 191, 192, 193.) Paris, Hermann, 1935. 42+52+23+42 pp.

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SASULY (M.). Trend analysis of statistics. Theory and technique. Washington, Brookings Institution, 1934. 14+421 pp.

SAVINE (S. A.). Ouvrages sur la théorie de l'élasticité. Fascicule 1. Harbin, Harbin Polytechnic Institute, 1934.

SCHNEIDER (E.). Entwicklungsgeschichte der physikalischen Weltanschauung. Berlin, Wegweiser Verlag, 1934. 322 pp.

SMITH (A. W.). Electrical measurements in theory and application. 3d edition. New York and London, McGraw-Hill, 1934. 18+413 pp.

STANLEY (L. C.). Surveying for the drawing office. London, Draughtsman Publishing Company, 1934. 51 pp.

TUPLIN (W. A.). Torsional vibration: elementary theory and design calculations. London, Chapman and Hall, 1934. 18+320 pp.

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WERKMEISTER (P.). Vermessungskunde. 3te Auflage. Band 3: Trigonometrische und barometrische Höhenmessung, Tachymetrie und Topographie. (Sammlung Göschen, Band 862.) Berlin, de Gruyter, 1934. 144 pp.

WILLIAMS (G. B.). The flow of water in pipes, sewers and channels, over weirs, and off catchments. London, Chapman and Hall, 1934. 76 pp.

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

ANN ARBOR, MICHIGAN, SUMMER MEETING AND COLLOQUIUM, September 10-14, 1935.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than August 6. Abstracts received by July 6 will appear in the July issue of the BULLETIN. Professor H. S. Vandiver will deliver a series of colloquium lectures on *Fermat's last theorem and related topics in number theory.* By invitation of the Program Committee, Professor G. Y. Rainich will deliver an address entitled *Product integrals,* and Professor G. T. Whyburn an address entitled *On the structure of continua.*

NEW YORK CITY, October 26, 1935.

Abstracts must be in the hands of Associate Secretary J. R. Kline, Low Memorial Library, Columbia University, New York City, not later than September 28. Abstracts received by the Secretaries by September 6 will appear in the September issue of this BULLETIN. By invitation of the Program Committee, Professor J. L. Walsh will deliver an address on *Interpolation by rational functions.*

CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA, November 30, 1935.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than November 2. All these will appear in the November issue of this BULLETIN.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or Low Memorial Library, Columbia University, New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1934, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, Low Memorial Library, Columbia University, New York City.

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